Физика систем с мотивацией: Проблемы моделирования человеческих действий

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Durkheim-Weber Dilemma

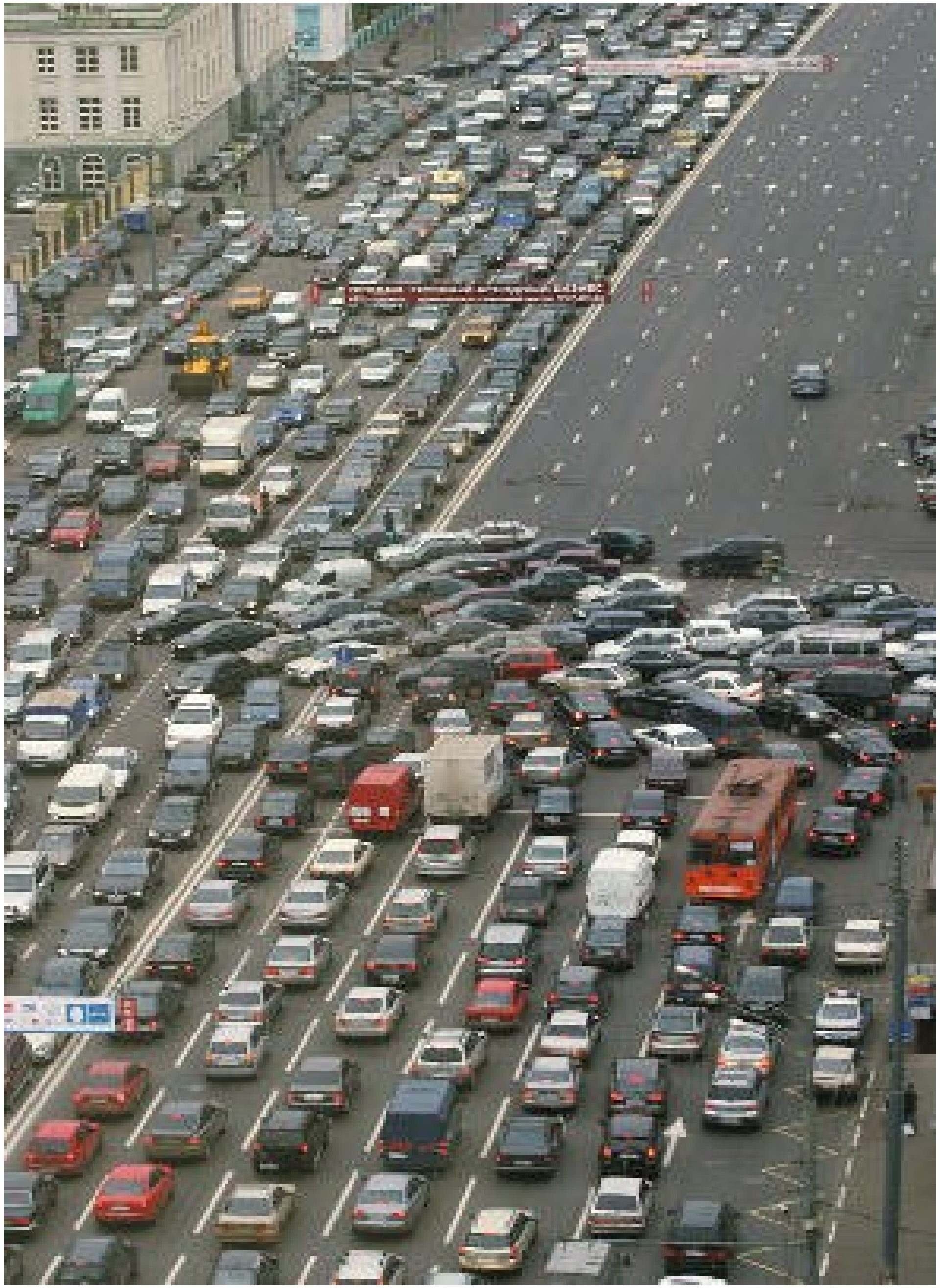
E. Durkheim: Math. & Physics are useful in Social Sciences

M. Weber: Math. & Physics are inapplicable to Social Sciences

Overcoming the Durkheim-Weber dilemma by reformulating it:

- *Durkheim's horn:* There is a wide class of phenomena observed in individual behavior of humans as well as social systems that admit mathematical description dealing with general laws independent of individual features of humans
- *Weber's horn:* New notions and mathematical formalism should be developed in addition to ones inherited from physics and applied mathematics

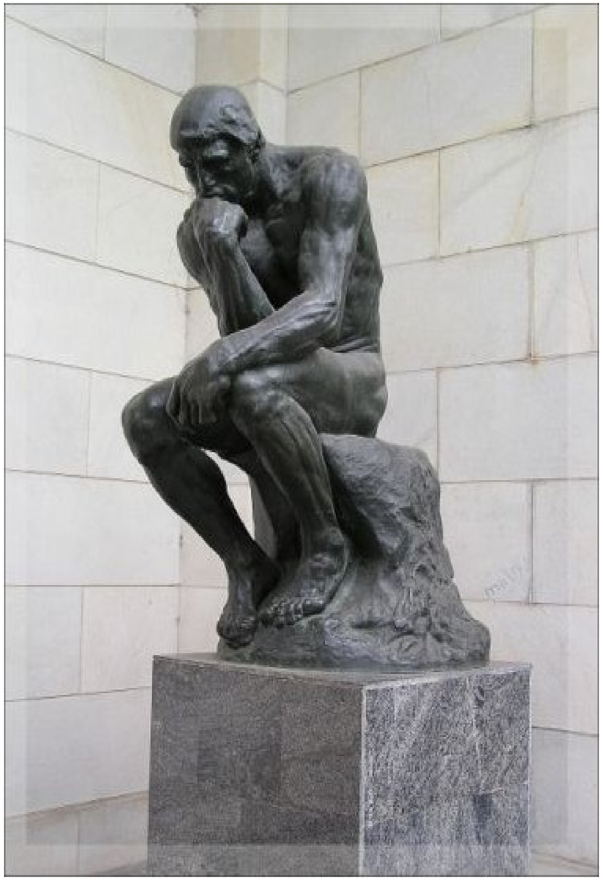
social systems with cooperative dynamics



social systems with cooperative dynamics

regular actions of humans

Systems with self-averaging & Characteristic element



- Final goal: what elements want
- Strategy of behavior:
  how elements are going to act:
  - perception
  - imagination
  - prediction
- Learning:
  how elements improve their behavior
- Social and moral norms
- Culture

the notion of
characteristic element:

- Regular properties
- Individuality as random factor

Ceteris paribus laws (laws with exceptions)

Multilevel nature of humans being:

Latin: ceteris paribus => with other things being the same

Our case:

Exclusive => non-controllable & non-disturbing

indefinite => unknown & unrecognized factors

ceteris paribus laws

Systems with cooperative or reproducible dynamics:

Holistic ceteris paribus laws

Nomological machines

Nancy Cartwright (1989)

A nomological machine is union of

- a fixed arrangement of components with stable capacities
- in a stable environment

that withing repeated operation reproduces the same kind of regular behavior described by a given scientific law (CP-law) (Cartwright, 1999)

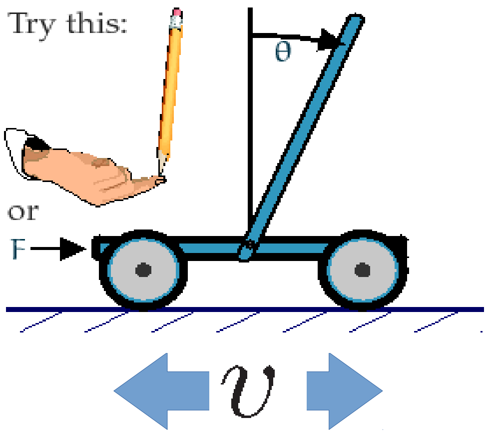
Examples: Pendulum, Solar System, etc.

Problem: Amount of nomological machines ?

How to merge nomological machines?

Examples of nomological machines

Stick balancing

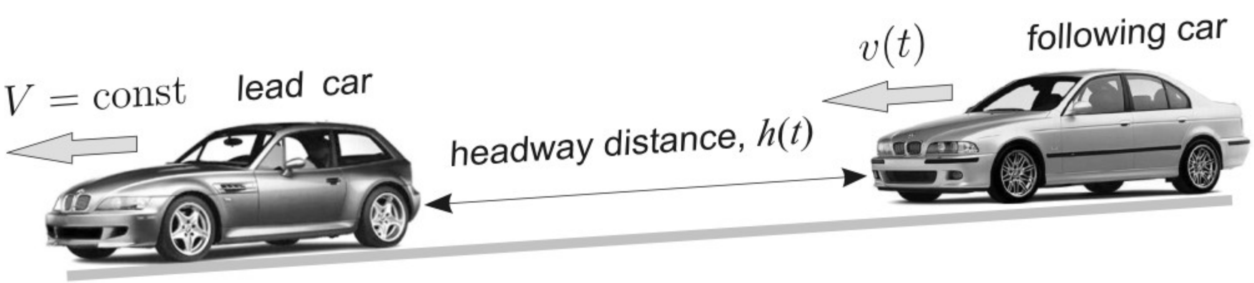


Model ?

$$\ddot{\theta} - \frac{1}{T^2}\sin\theta + \frac{1}{\tau}\dot{\theta} = -\mathcal{R}(\theta,\dot{\theta})$$

$$\mathcal{R}(\theta,\dot{\theta}) = \beta(\theta - \theta_{\mathrm{th}})\left\{p_0\theta_{i-\tau} + p_1\dot{\theta}_{t-\tau}\right\}$$

Car following



Model (Social force model) ?

$$\frac{dv}{dt} = a_{\mathrm{opt}}(h, v)$$

$$a_{\mathrm{opt}}(h, v) = \frac{1}{\tau_v}\left[v_{\max}\frac{h^2}{h^2 + D^2} - v\right]$$

Human Intermittent Control:
Dynamical Trap Theory &
Virtual Experiments on Stick Balancing

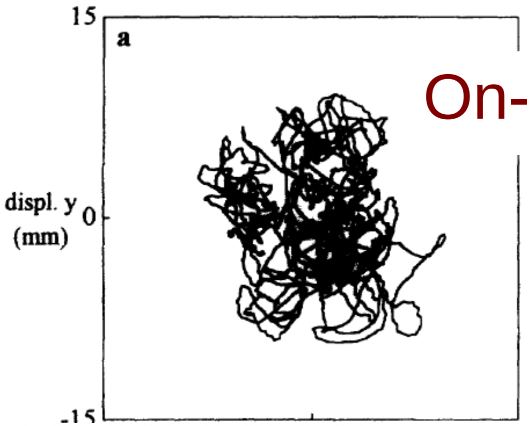
$$\ddot{\theta} - \frac{1}{T^2}\sin\theta + \frac{1}{\tau}\dot{\theta} = -\mathcal{R}(\theta, \dot{\theta})$$

$$\mathcal{R}(\theta, \dot{\theta}) = \beta(\theta - \theta_{\text{th}}) \left\{ p_0 \theta_{i-\tau} + p_1 \dot{\theta}_{t-\tau} \right\}$$

Characteristic properties of human response:

Discontinuous control by human actions

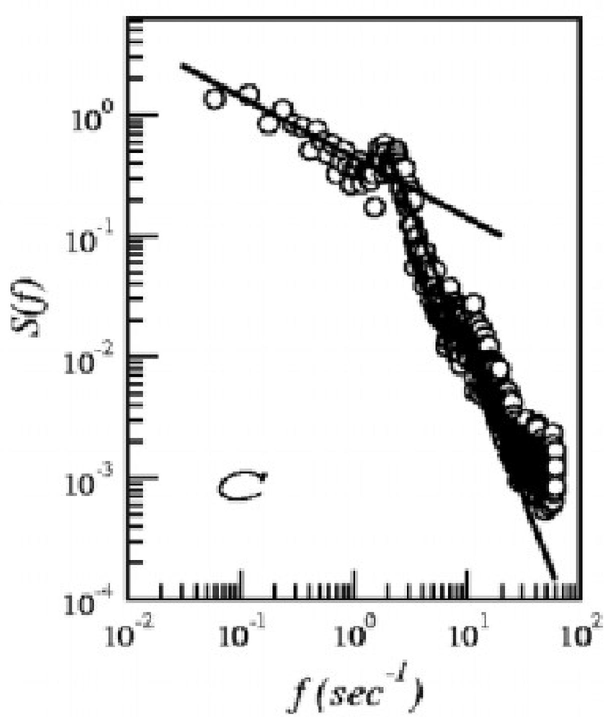
Empirical data of muscles behavior during body sway

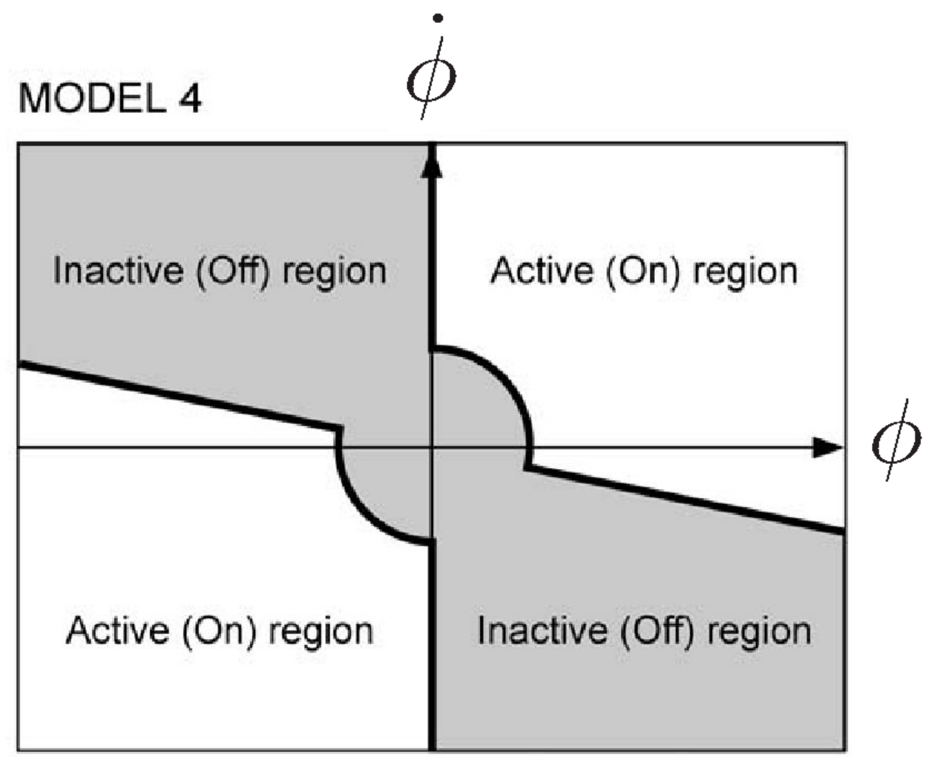


On-Off Intermittency

Act-and-Wait or
Event-Driven models

Different regions in the phase space $\{\phi, \dot{\phi}\}$





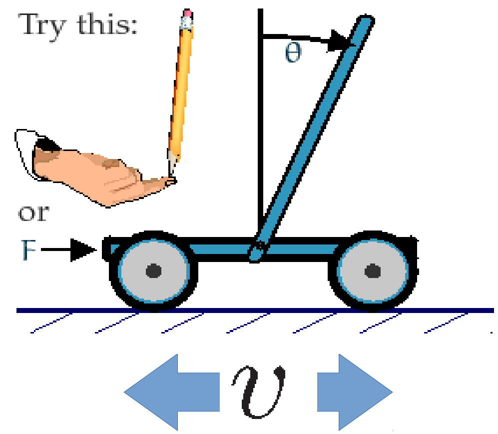
Virtual experiments with over-damped inverted pendulum

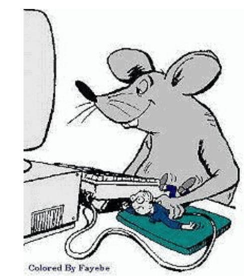
Mechanics:

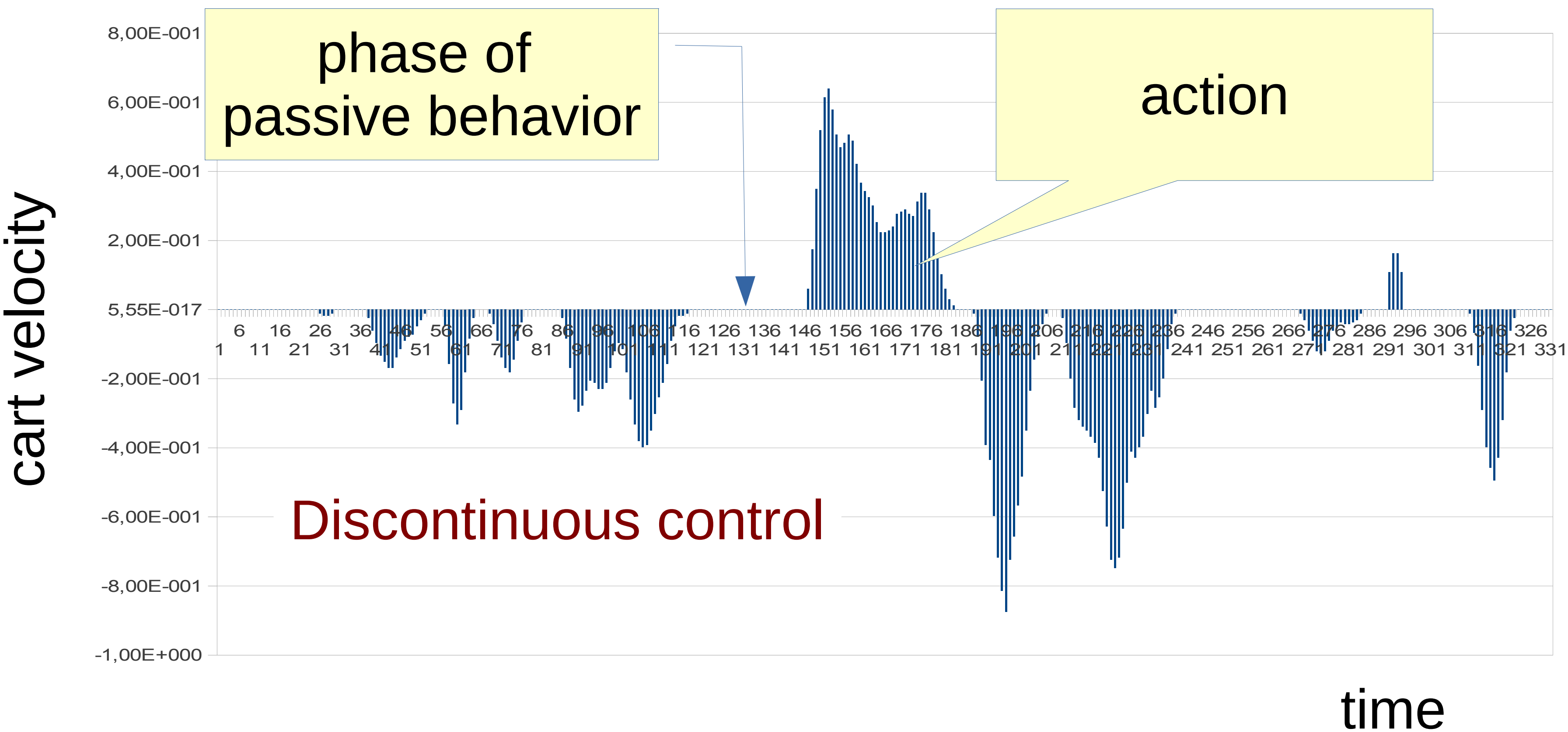
$$\tau\dot{\phi} = \sin\phi + \frac{\tau}{l}\cdot\vartheta\cdot\cos\phi$$

Human action:

$$\vartheta = U\{\phi, \dot{\phi}\}$$







Human Intermittent Control (*Modern paradigm*)

Human actions in control over unstable mechanical systems form

a sequence of alternate fragments of

- Passive phase: When the system state is rather close to the equilibrium the operator cannot recognize its deviation from the desired state and does nothing ("Status quo bias")

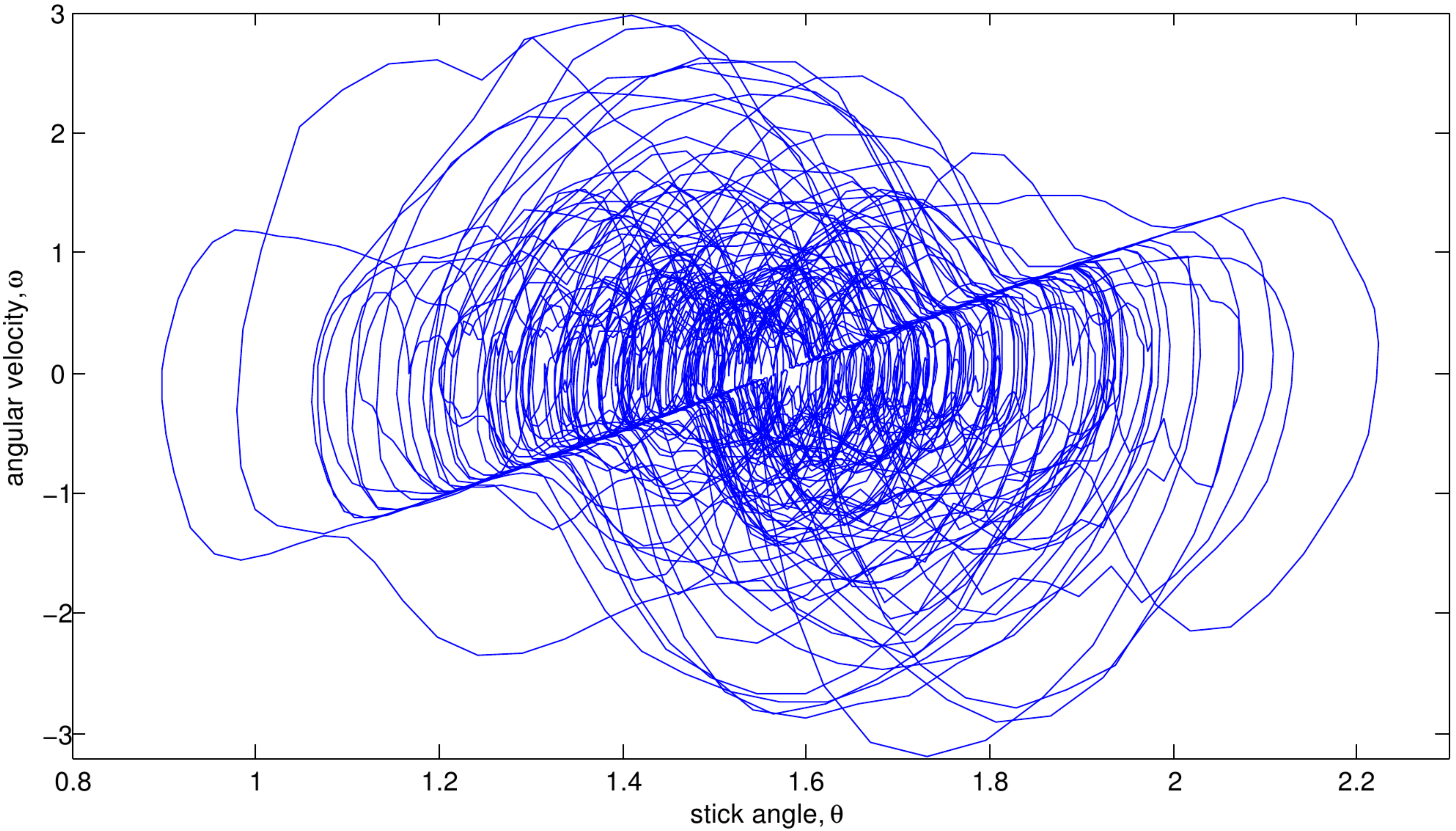
- Active phase: Open-loop control, during a given fragment of active phase the operator practically does not react to the current system dynamics

Two phase transitions:

Passive phase → Active phase | Active phase → Passive phase

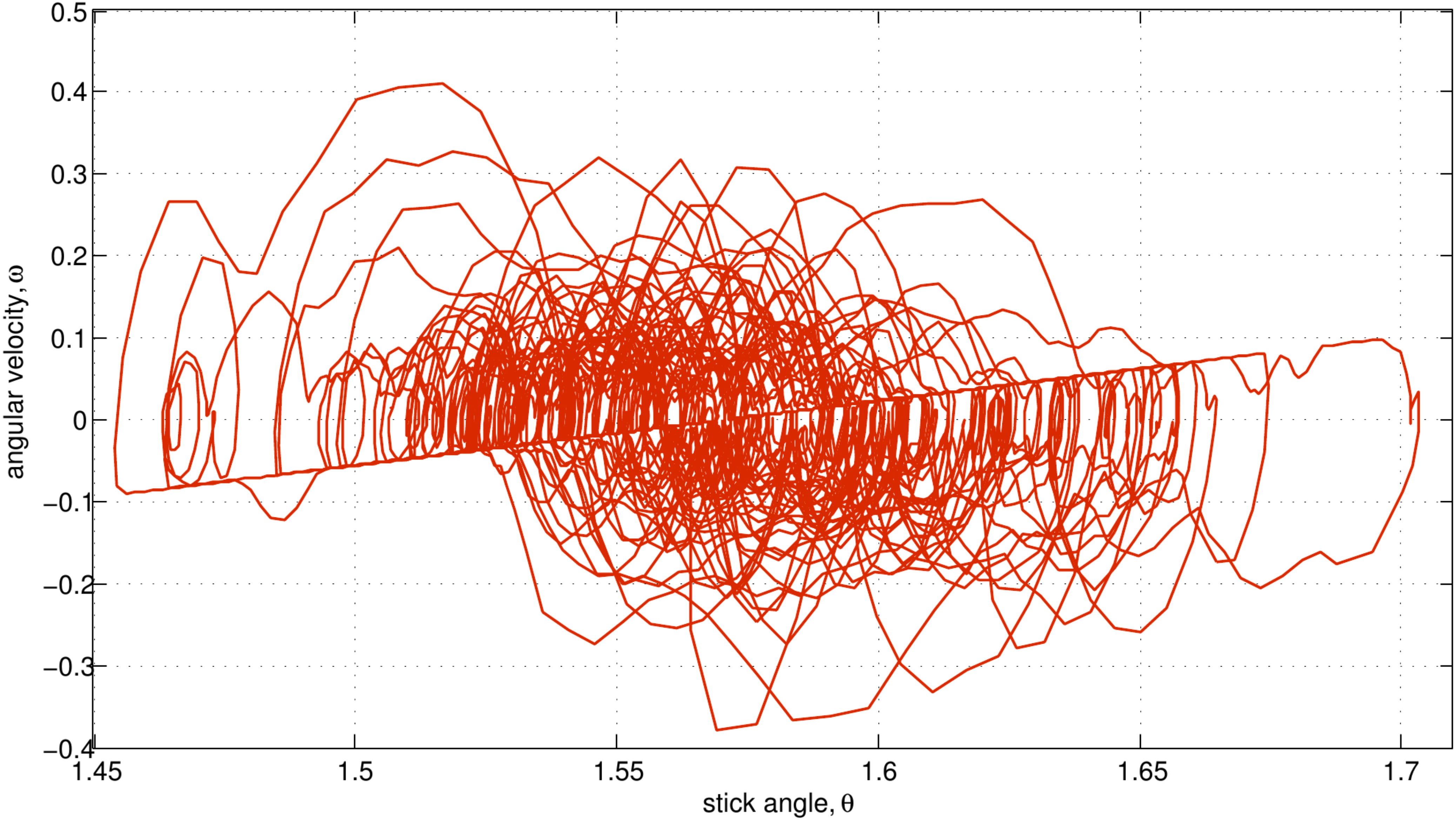
Virtual inverted pendulum:

Phase portrait: $\tau = 1 \text{ sec}$



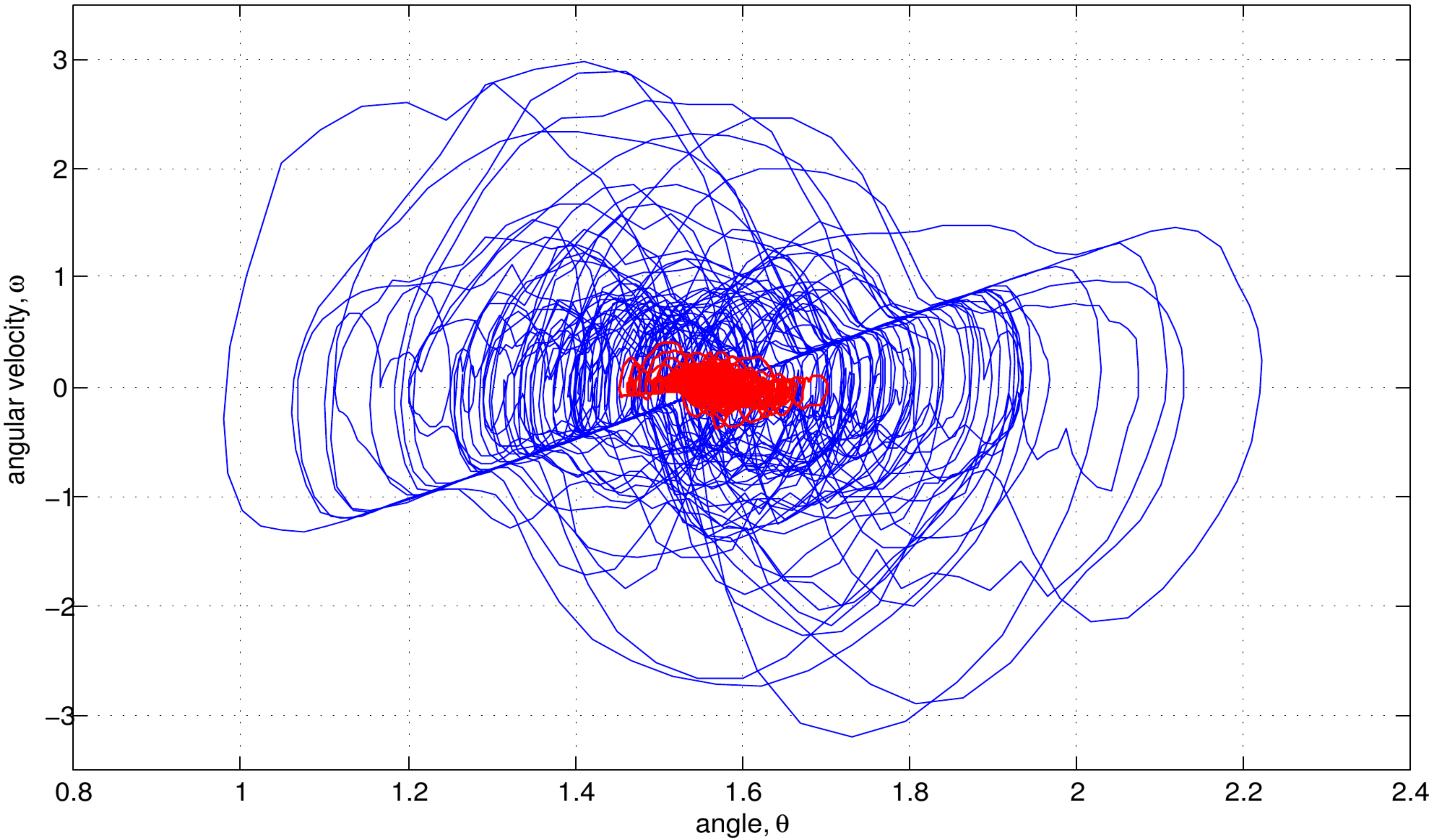
Virtual inverted pendulum:

Phase portrait: $\tau = 5 \text{ sec}$



Virtual inverted pendulum:

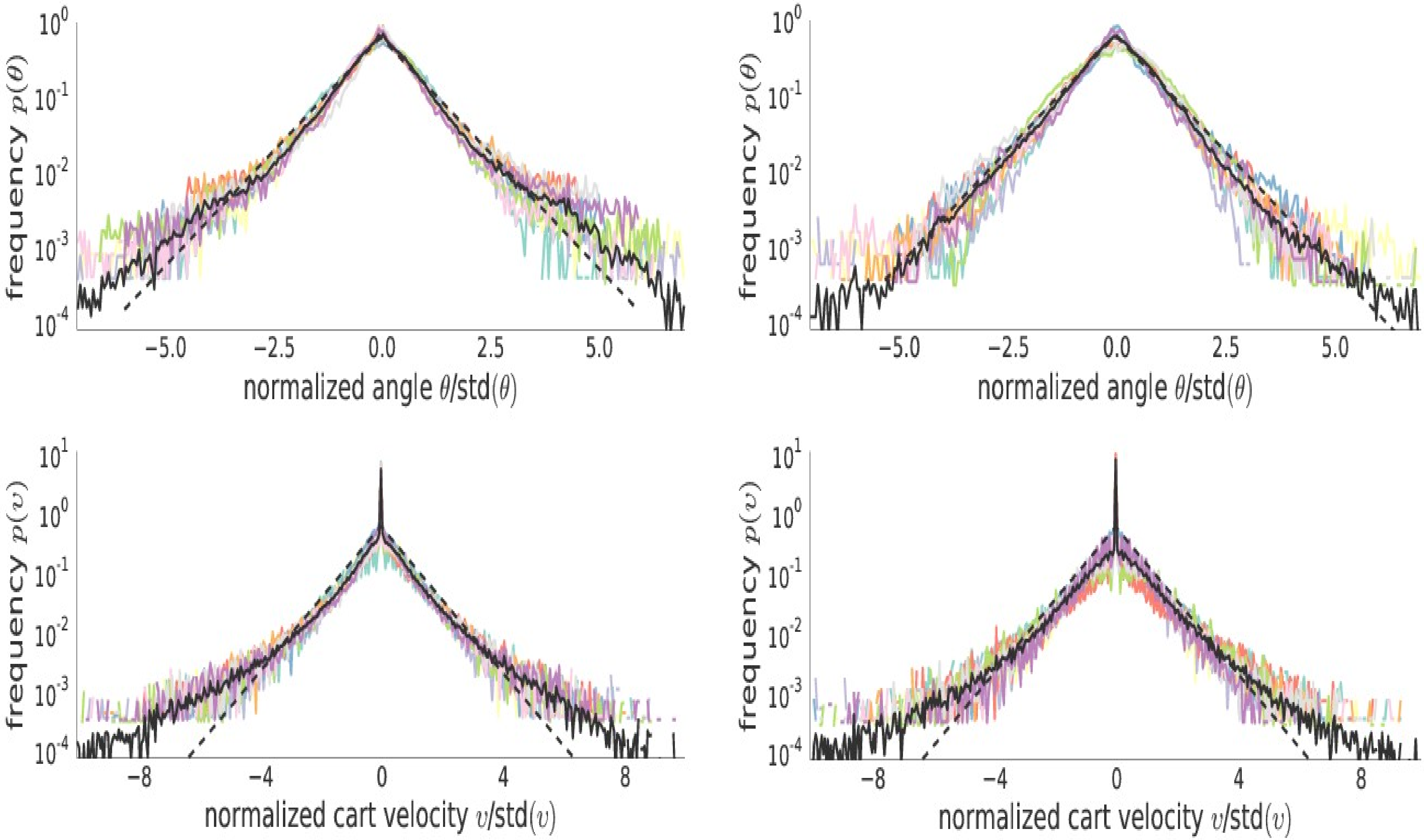
Phase portrait: $\tau = 1 \;\&\; 5 \text{ sec}$



Virtual inverted pendulum: Experimental data

“Fast” stick

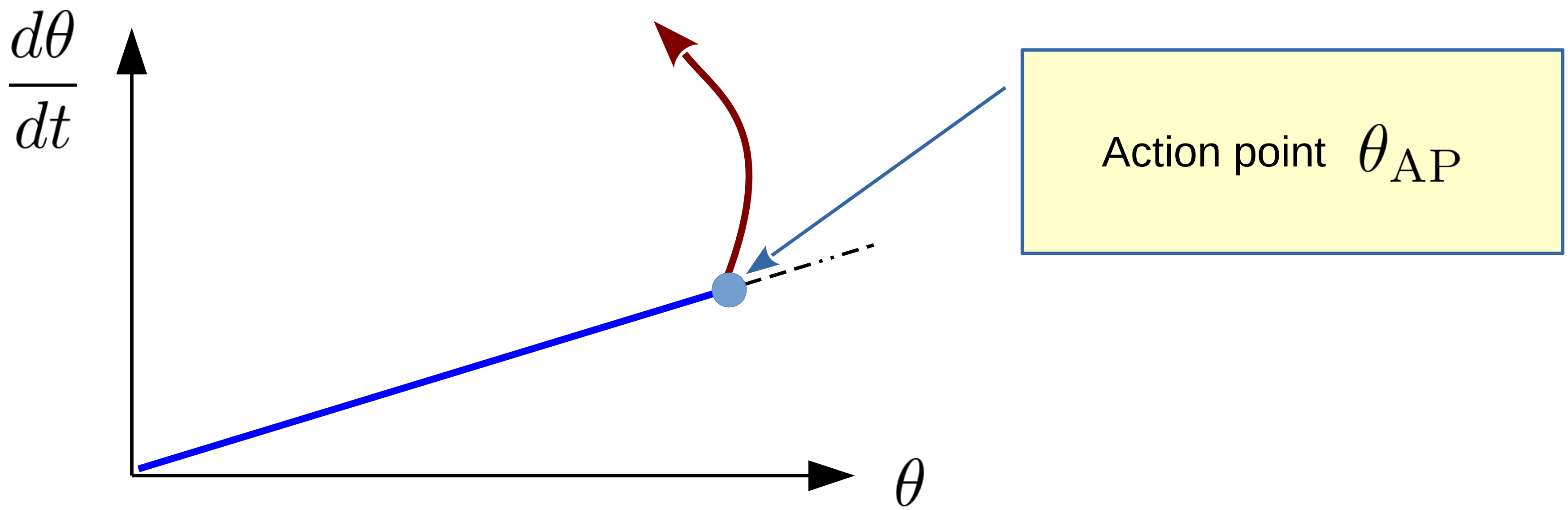
“Slow” stick



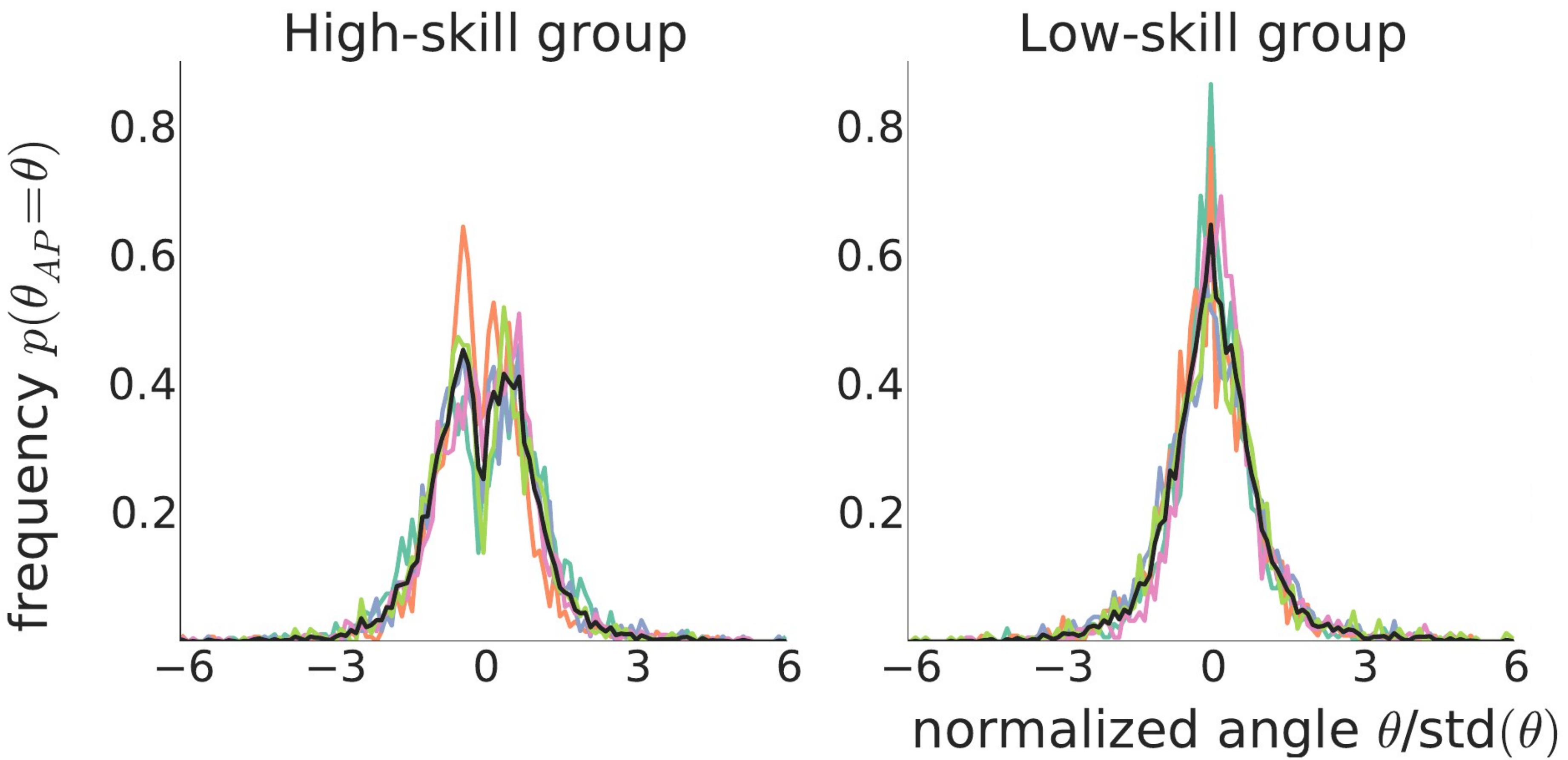
Phase transition: Passive phase → Active phase

Passive phase: The operator does nothing and "accumulates information" about the system state

The system dynamics is governed by its mechanical laws and can be described by the corresponding phase space. All the human factors such as a delay time (regular and probabilistic) can be incorporated into the concept of action points and their probabilistic properties:



Virtual inverted pendulum: Experimental data



phase space $\{x,y\}$

Human Bounded Rationality and Dynamical Trap Concept

if human perception were strictly rational

$$\tau \frac{dx}{dt} = P(x,y) \qquad \tau \frac{dy}{dt} = Q(x,y)$$

$$P(x,y) = 0 \qquad Q(x,y) = 0$$

stationary point

$$\{x_{\mathrm{st}}, y_{\mathrm{st}}\}$$

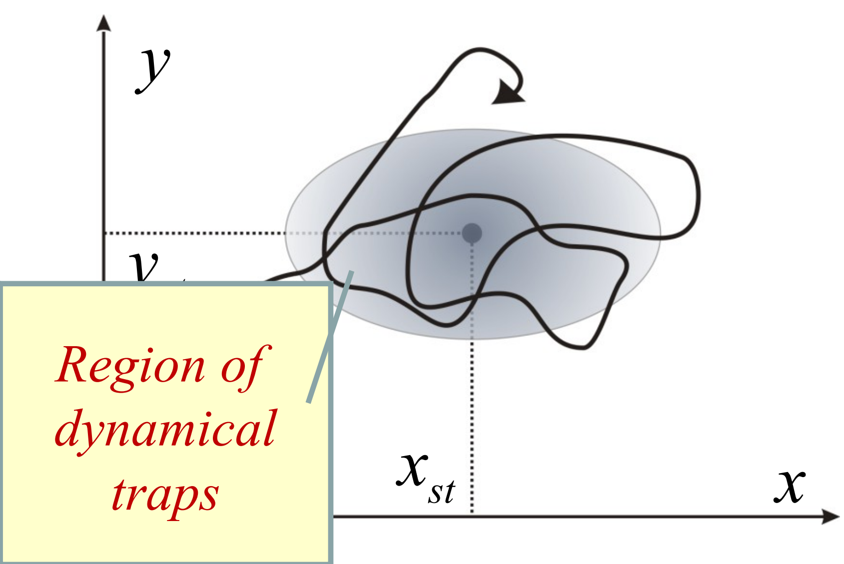
stable or unstable

Bounded Rationality $\{x,y\} \to \{x_{\mathrm{st}}, y_{\mathrm{st}}\}$

$$\tau(x,y) \to \infty$$

$$P(x,y) \to P(x,y) + \epsilon_x(x,y)\xi_x(t)$$

$$Q(x,y) \to Q(x,y) + \epsilon_y(x,y)\xi_y(t)$$

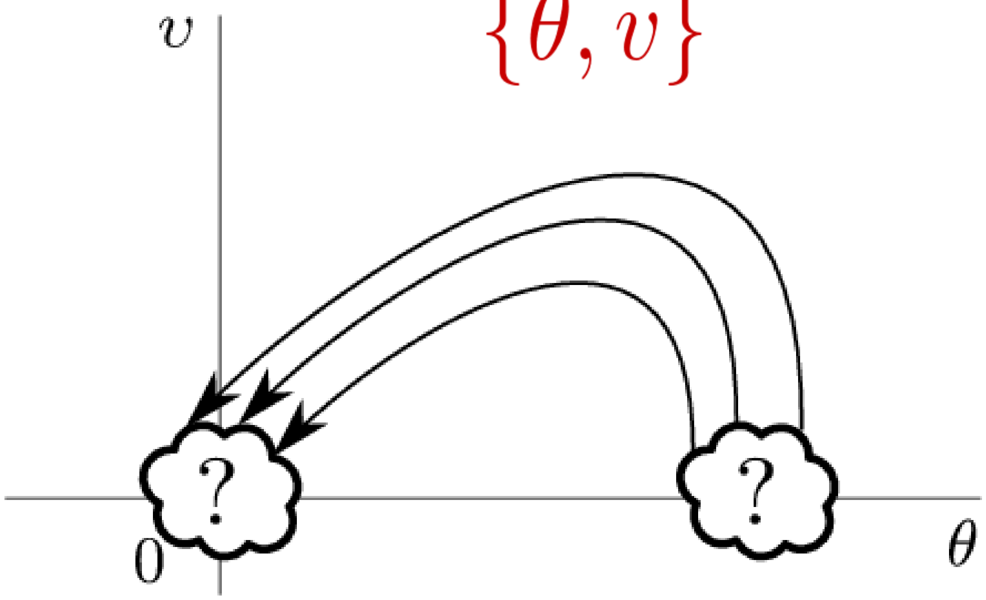


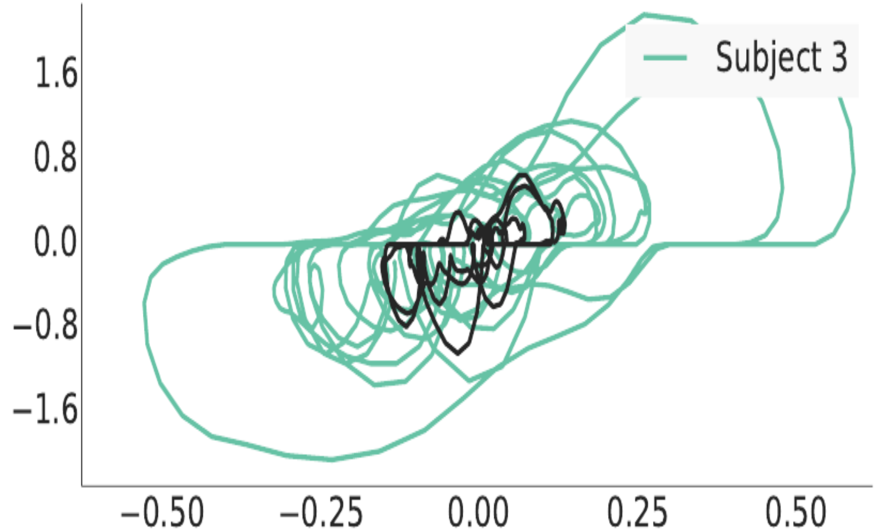
person just cannot "see" the point $\{x_{\mathrm{st}}, y_{\mathrm{st}}\}$

Phase transition: Active phase → Passive phase

Active phase: Open loop control: Extended Phase Space

$\{\theta, v\}$





Rational approximation: minimize subject to

$$\mathcal{F}\{v\} = \int_t^\infty \left[\frac{\tau^2}{2l^2}\left(v^2 + \tau_m^2 \dot{v}^2\right) + \frac{\theta^2}{2\theta_m^2}\right] dt'$$

$$\tau\dot{\theta} = \theta - \frac{\tau}{l} v$$

Dynamical Trap Model

System mechanics:

$$\tau \frac{d\theta}{dt} = \theta - \frac{\tau}{l} v$$

Effective noise for Passive Phase → Active Phase

Human actions:

$$\frac{dv}{dt} = \Omega(v)(\gamma\theta - \sigma v) + \epsilon\xi(t)$$

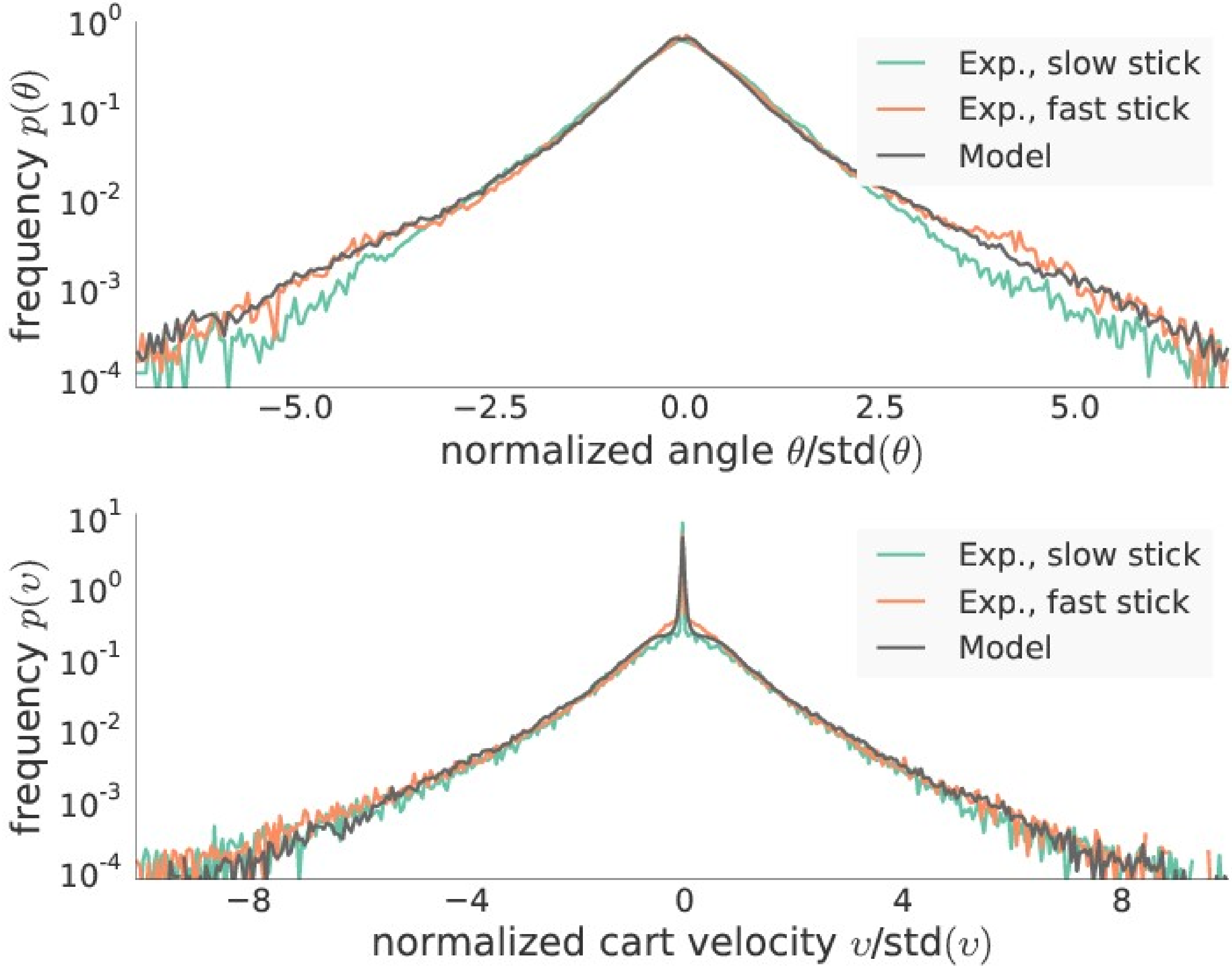
Active Phase → Passive Phase

Open Loop Control

$$\Omega(v) = \frac{v^2}{v^2 + v_{\text{th}}^2}$$

Dynamical Trap Factor for the Cart Velocity

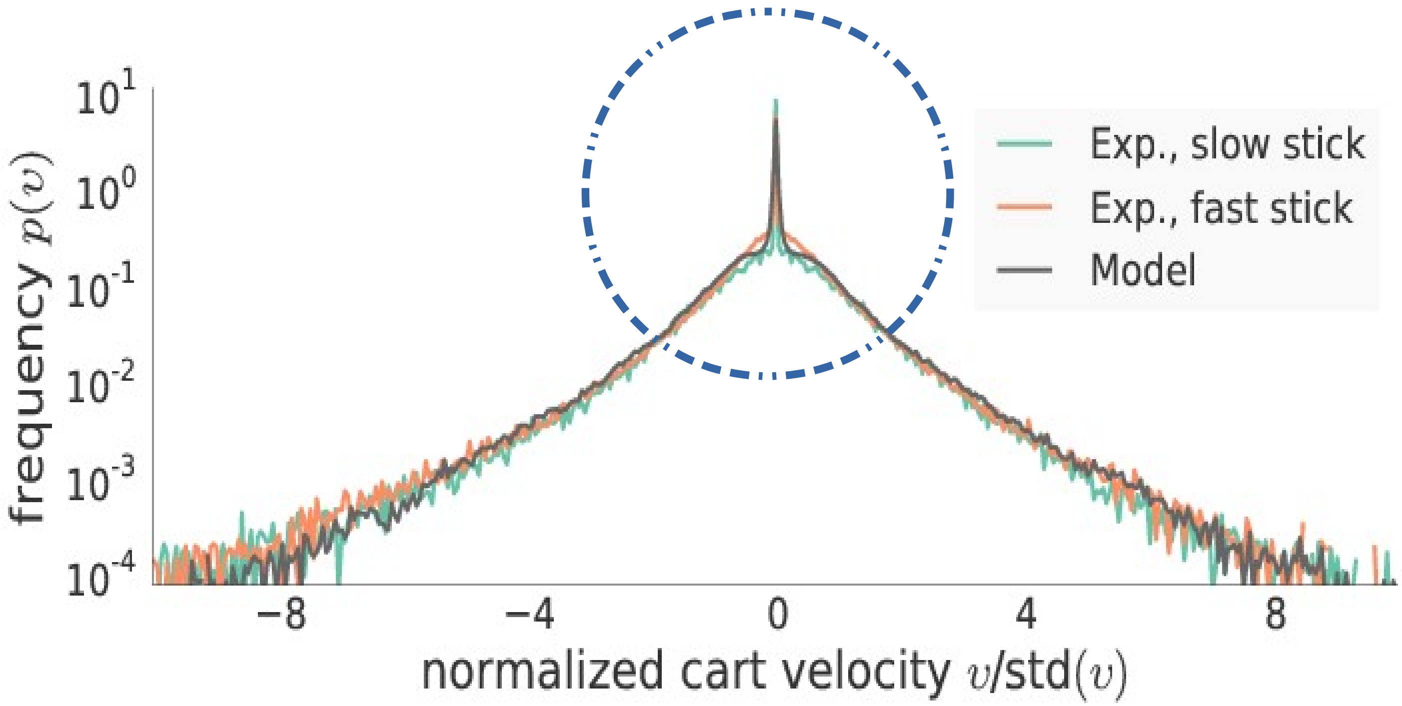
Model vs. Experiments: Angle & Cart velocity



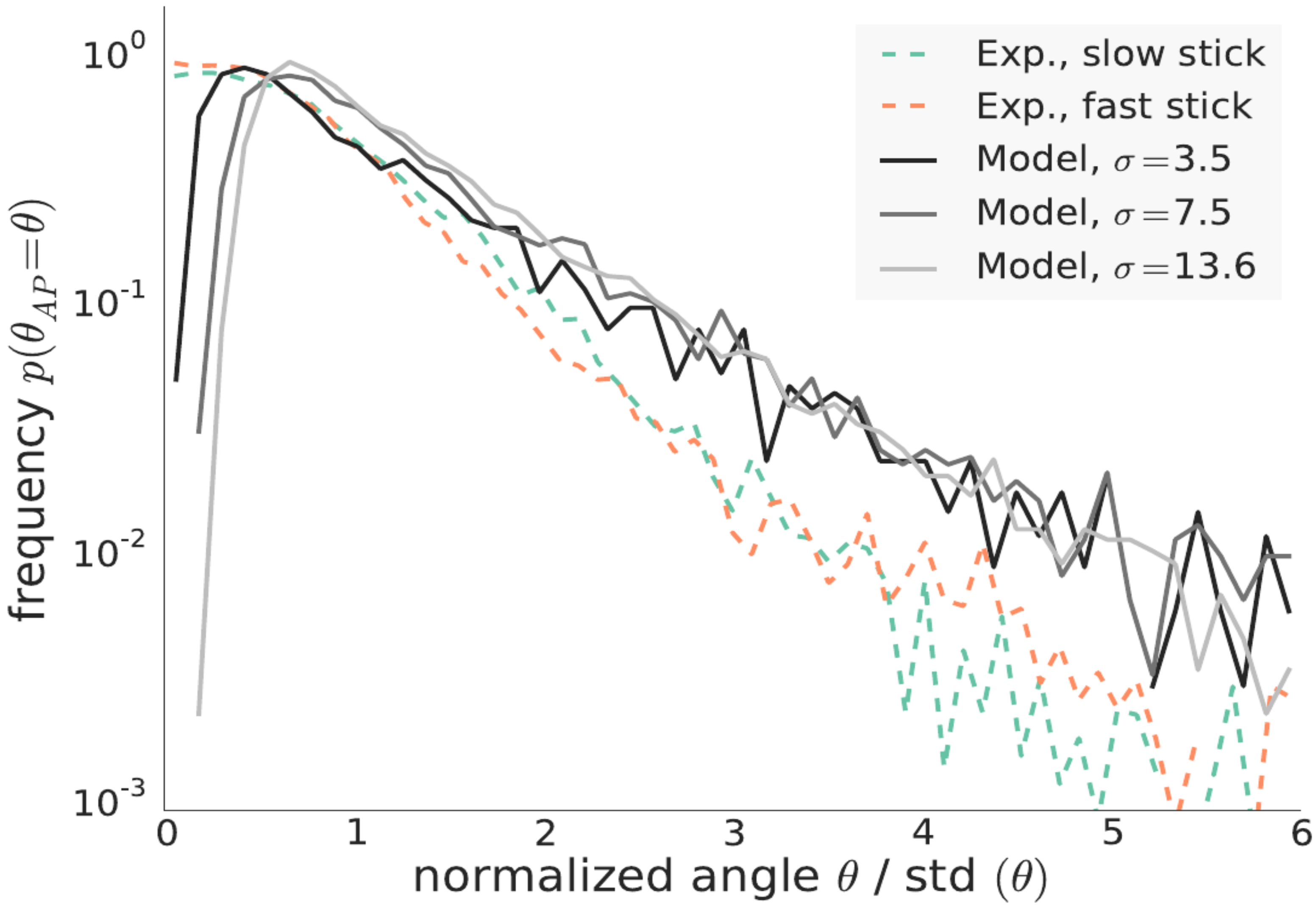
Characteristic Property of Intermittent Control

Peak of the cart velocity distribution: ⇒

- Measure of the passive phase amount
- Cart velocity as additional phase variable describing human actions

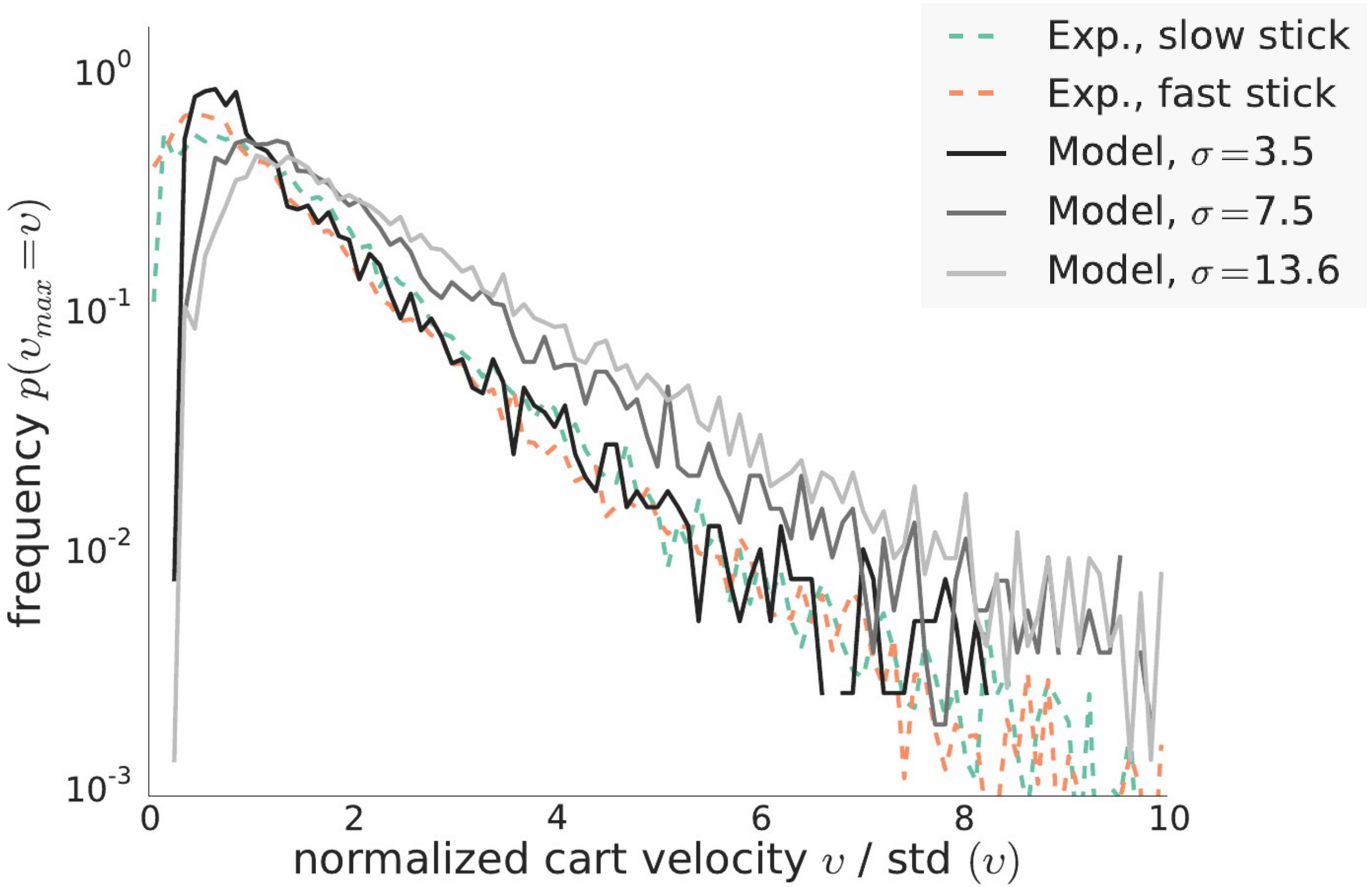


Model vs. Experiments: Action Points



(a) Action point distribution.

Model vs. Experiments: Maximum velocity



(b) Peak velocity distribution.

Conclusion to Over-Damped Stick Balancing Experiments

Description of the dynamics of

the balancing of over-damped pendulum

doest not belong to the Newtonian mechanics paradigm.

The dynamics of such human intermittent control is based on a sequence of alternate phase transitions of probabilistic nature

Active phase ⇔ **Passive phase**

Open-loop control requiring the extended phase space:

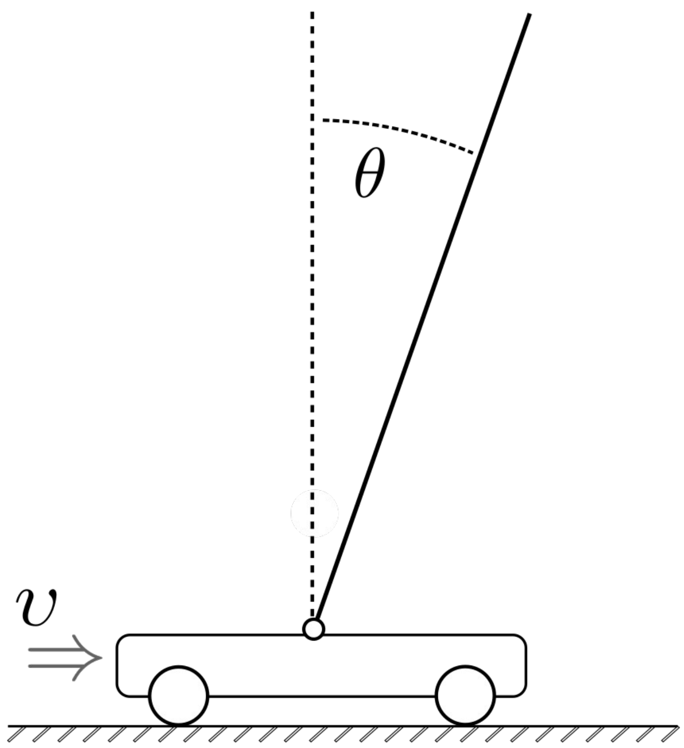
stick angle $- \theta$

cart velocity $- v$

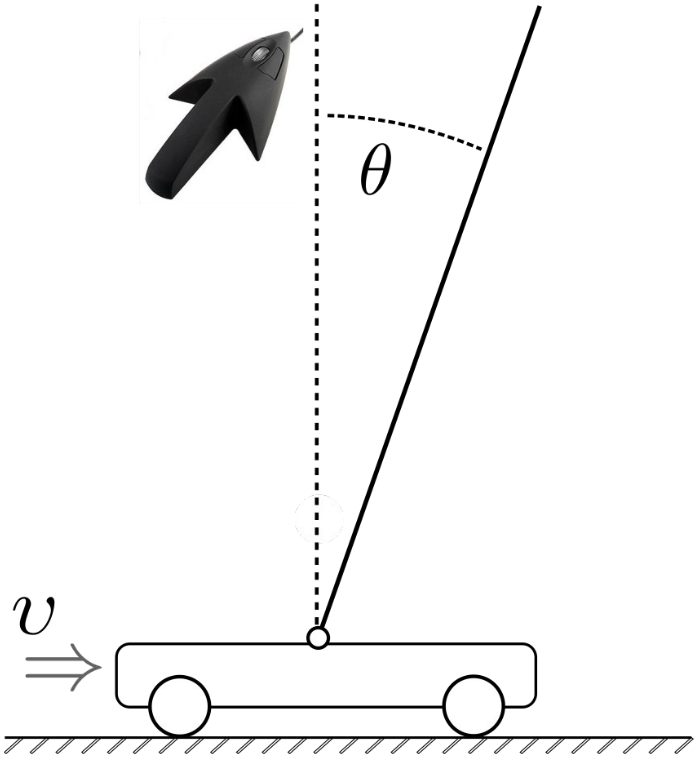
System dynamics is described by the standard phase space

Stick balancing: two nomological machines

no mouse

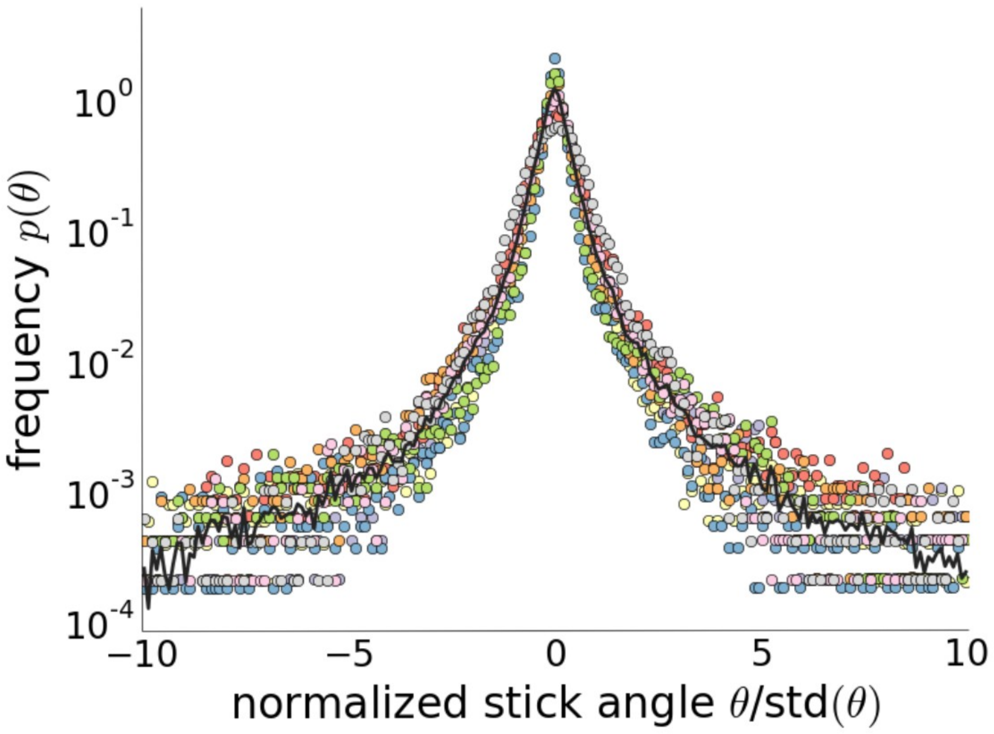


mouse is visible

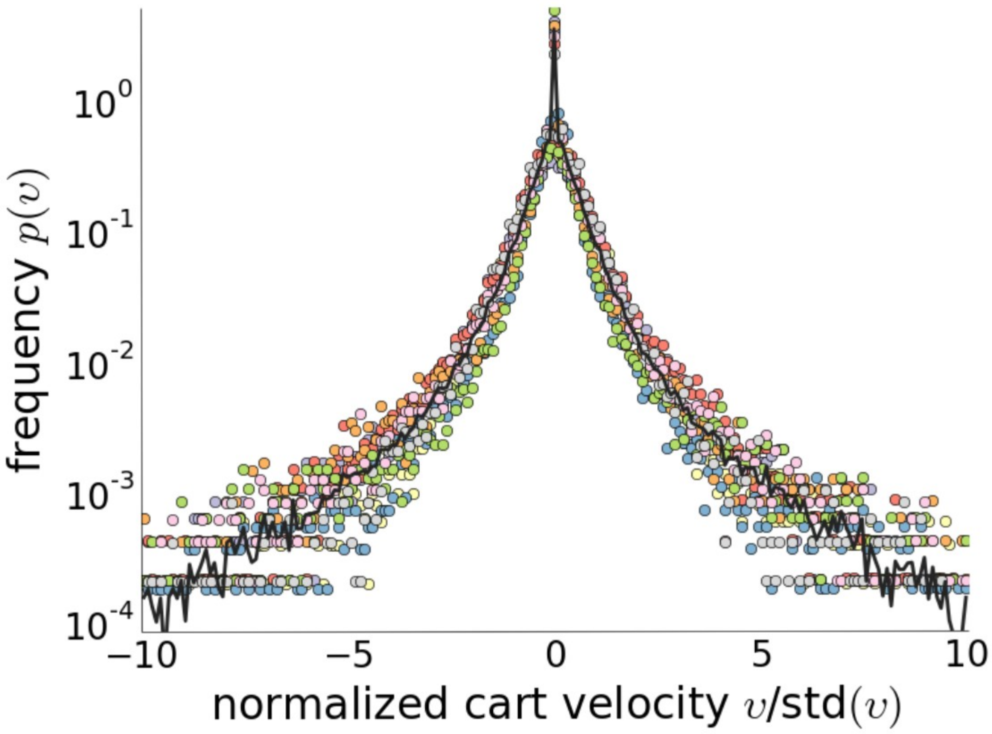


Visible mouse experiments, 8 subjects

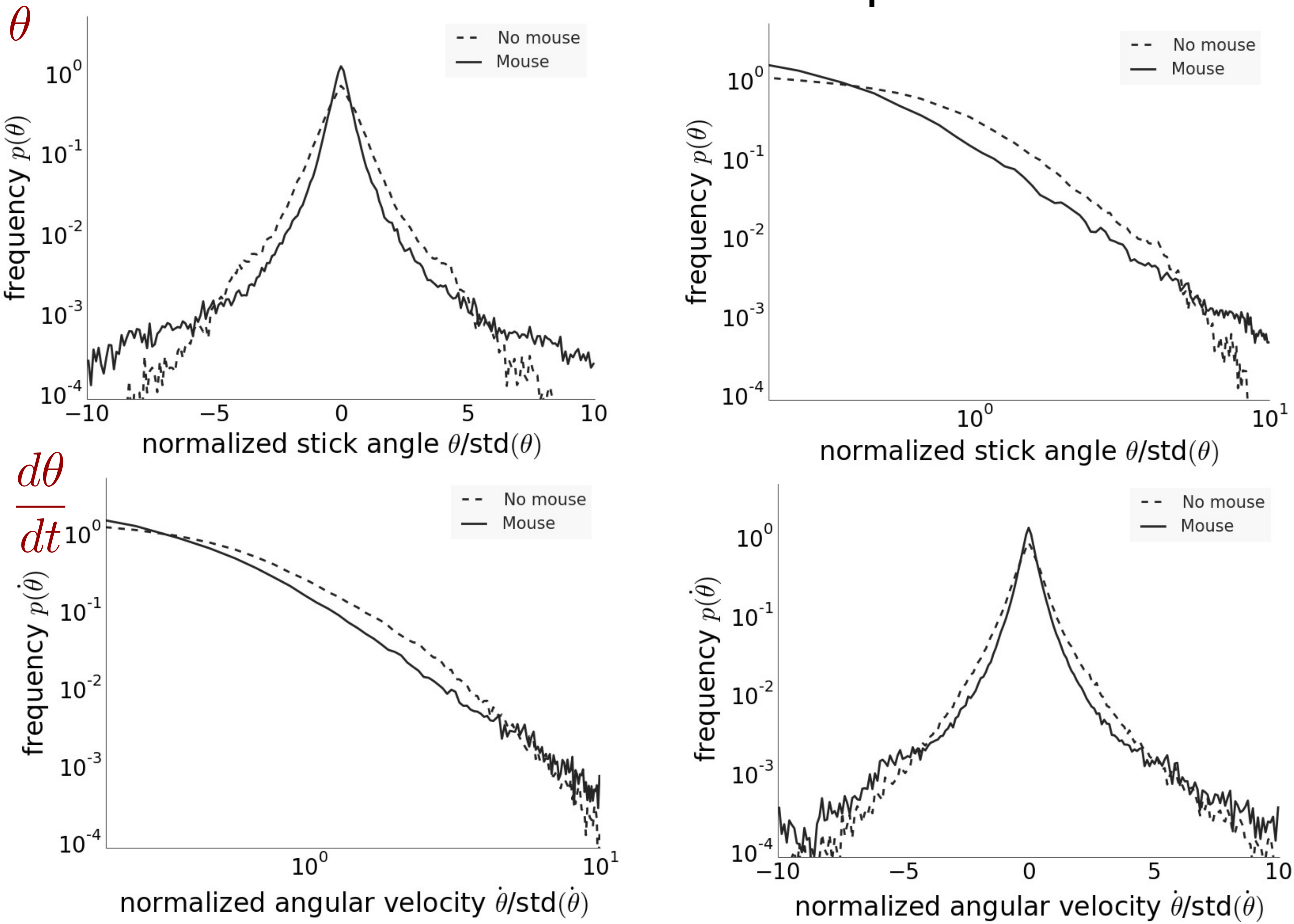
Angle distribution



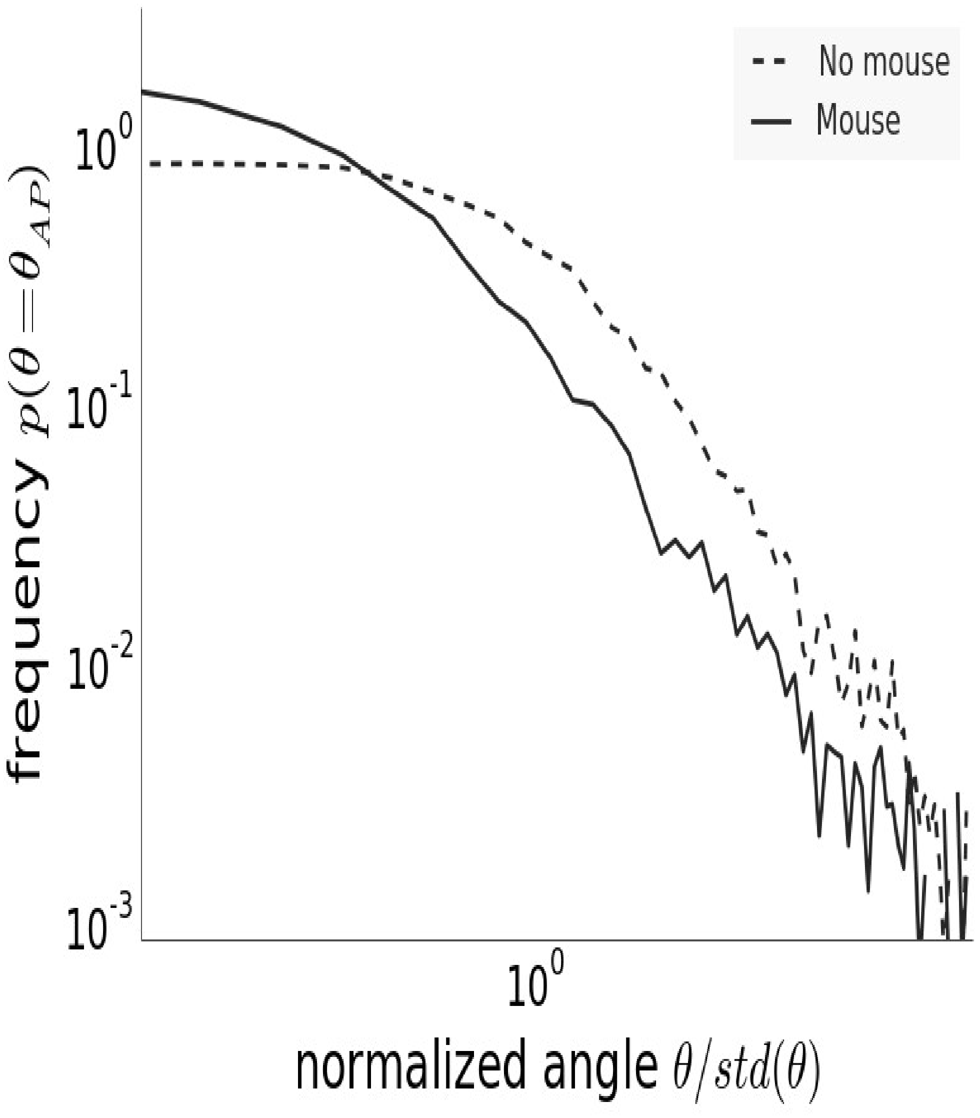
Car velocity distribution

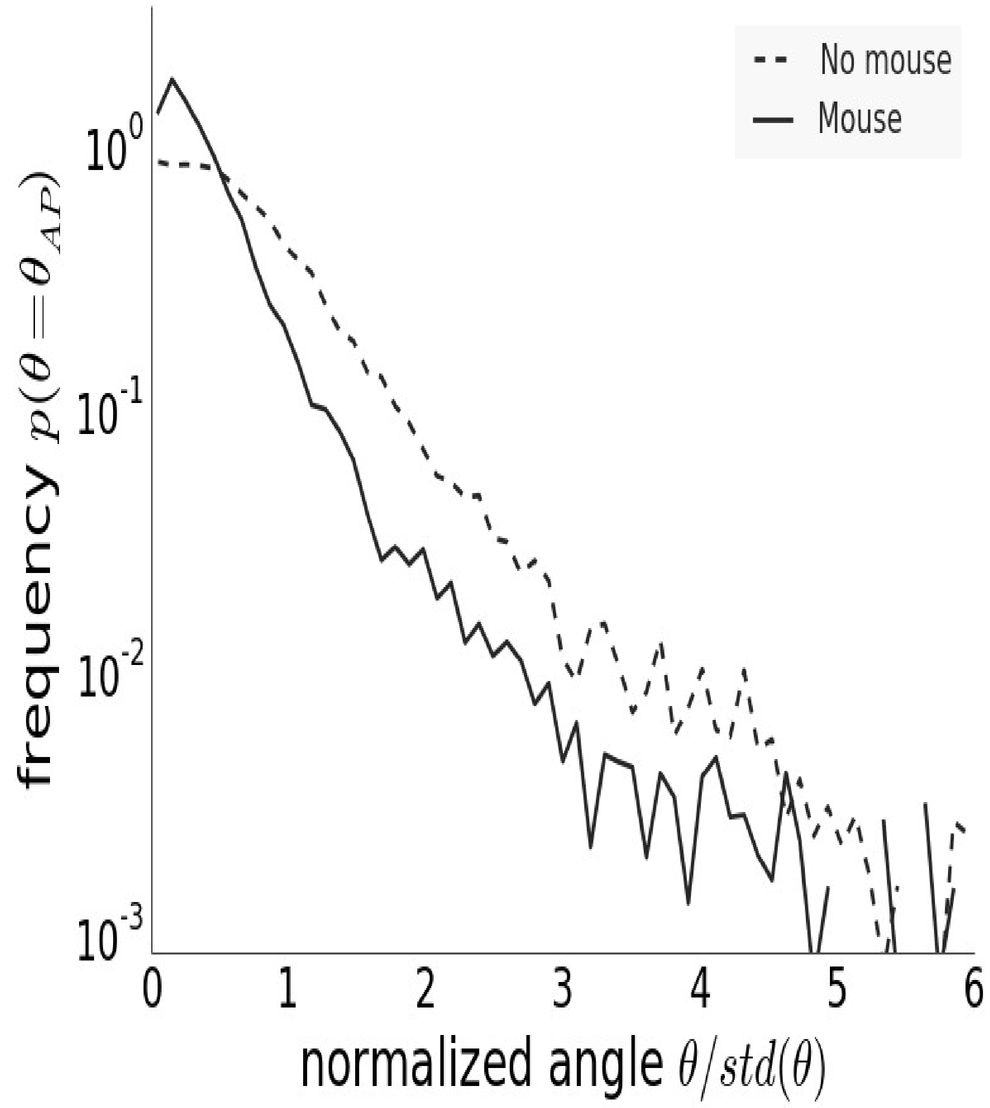


“Mouse – No-mouse” comparison

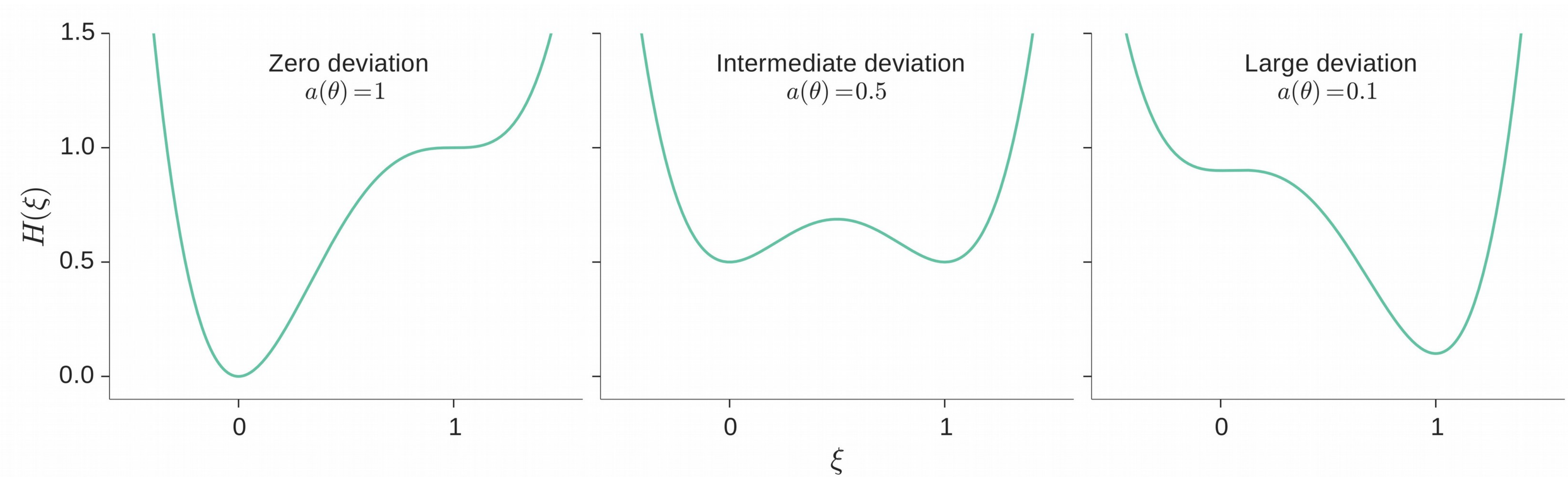


“Mouse – No-mouse” comparison: Action Points





Double-well dynamics of control activation



Governing equations

$$\dot{\theta} = \theta - \upsilon,$$

$$\dot{\upsilon} = \gamma\theta\xi - \sigma\upsilon,$$

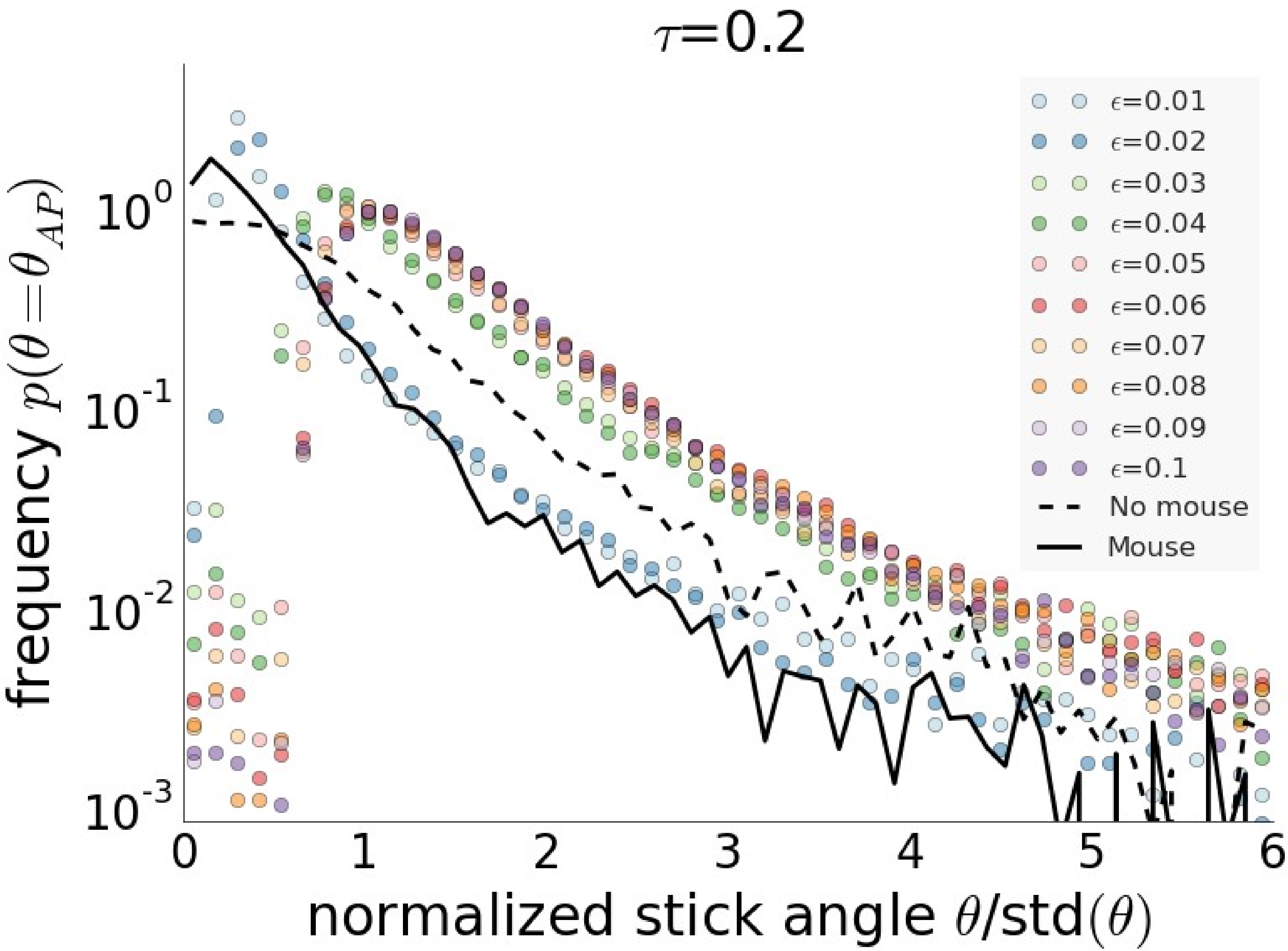
$$\tau\dot{\xi} = -\frac{\partial H}{\partial \xi} + \sqrt{\epsilon H}\zeta,$$

Model of potential

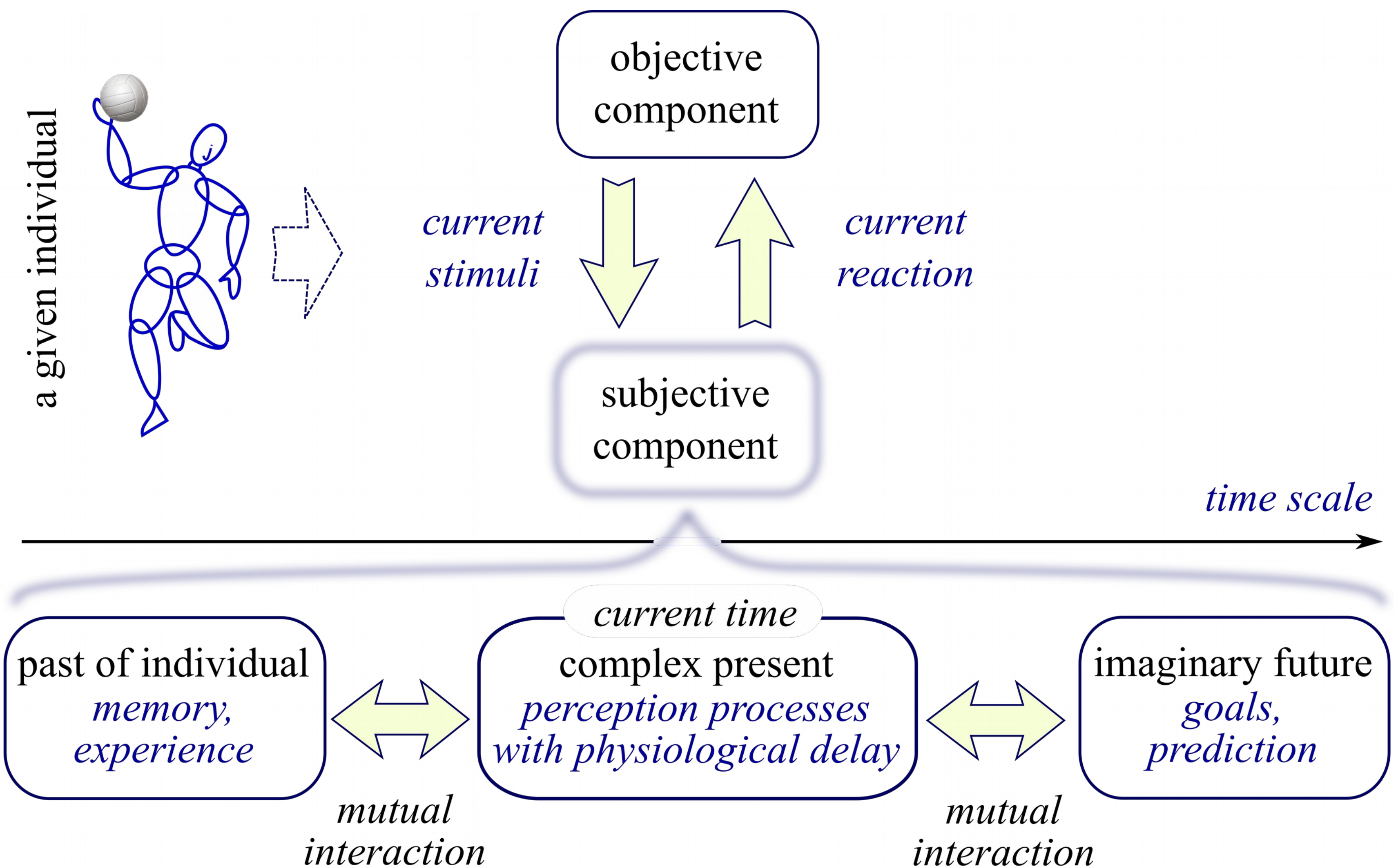
$$H(\xi, a(\theta)) = 3\xi^4 - 4(1+a)\xi^3 + 6a\xi^2 + 1 -$$

$$a(\theta) = 1/(1+\theta^2)$$

Double well potential:
Phase transition in the action point distribution



Description of human actions:
Two individual components



Car driving simulator (TORCS)

4 subjects with different driving skill

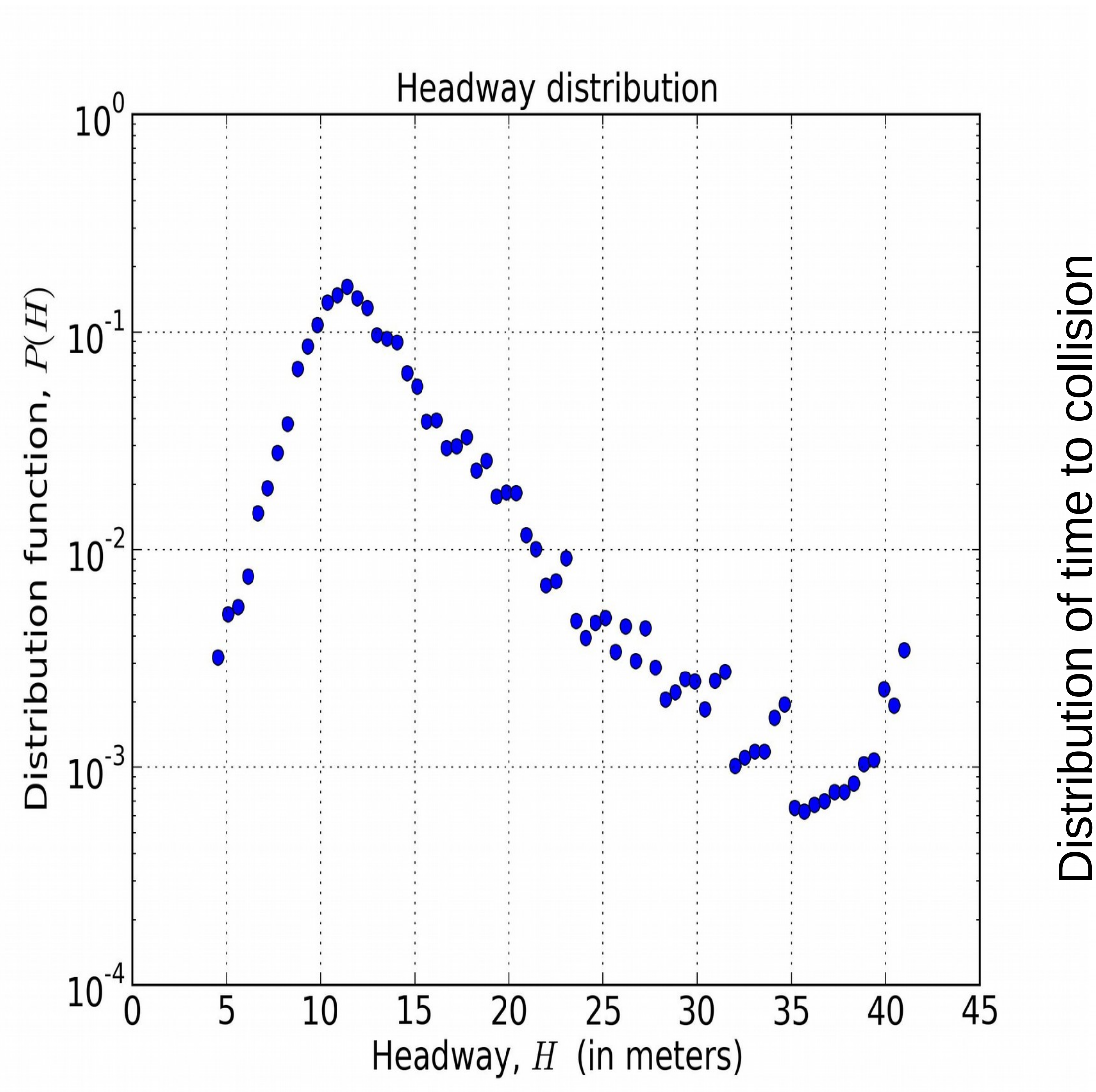
Car following setup

- follow a lead car without overtaking;
- keep a convenient headway without loosing the lead car

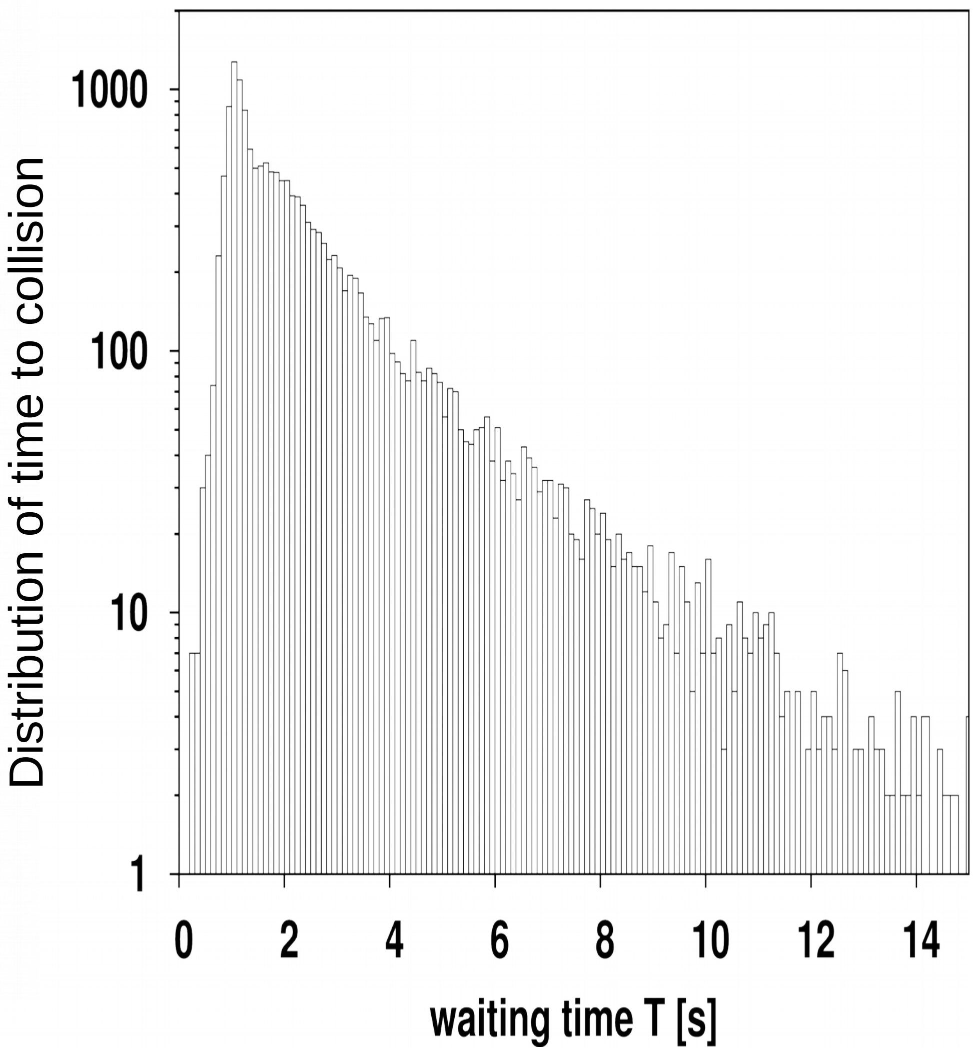
Duration of experiment: 30 minutes

Headway distribution (TORCS)

Data of virtual experiments

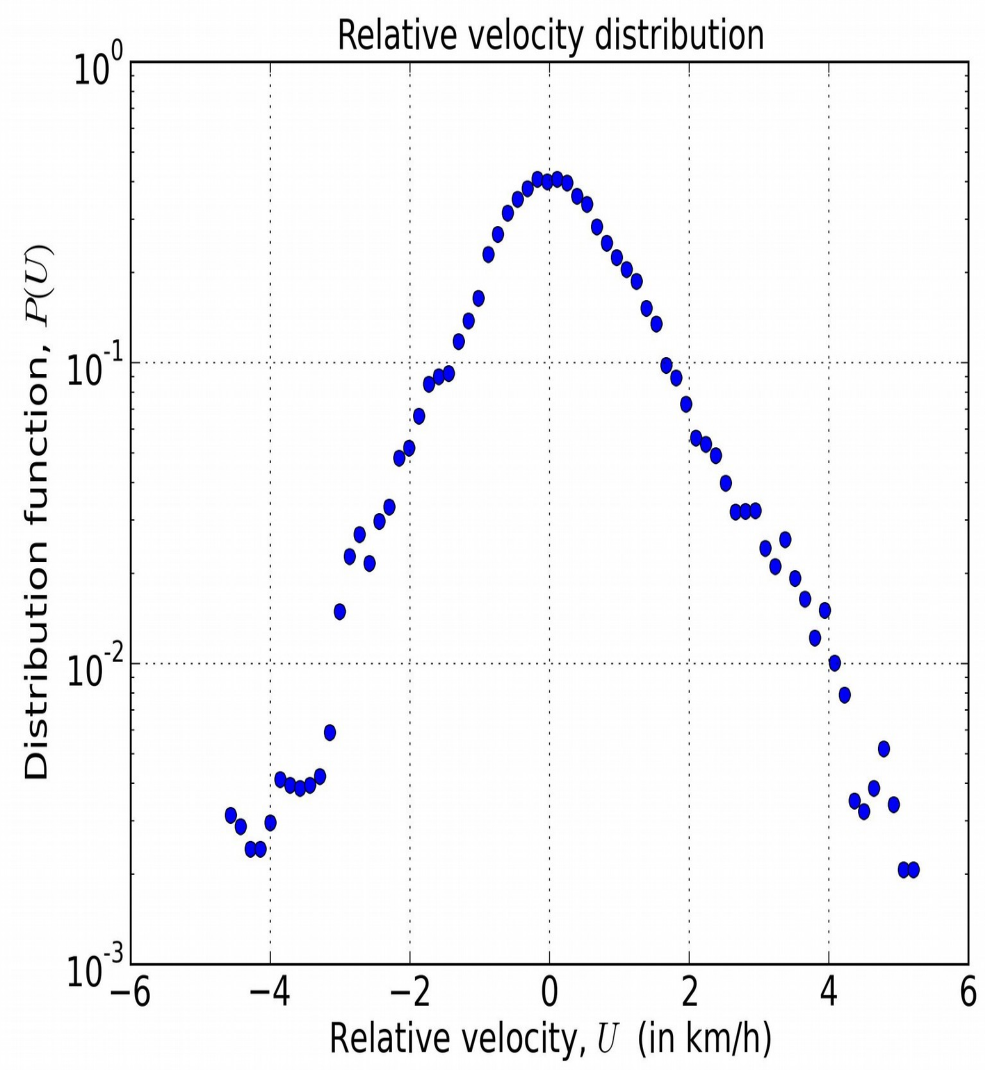


Data of real traffic

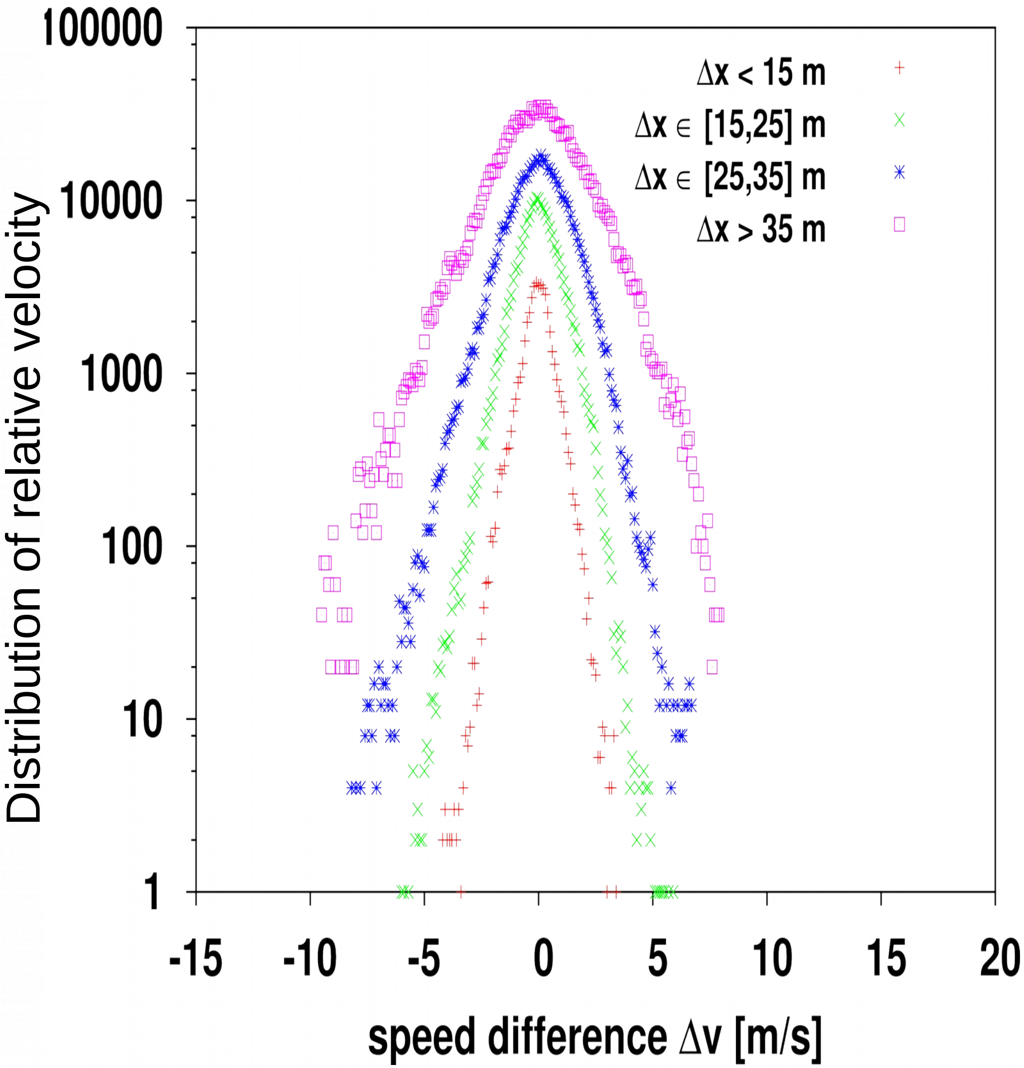


Velocity distribution (TORCS)

Data of virtual experiments



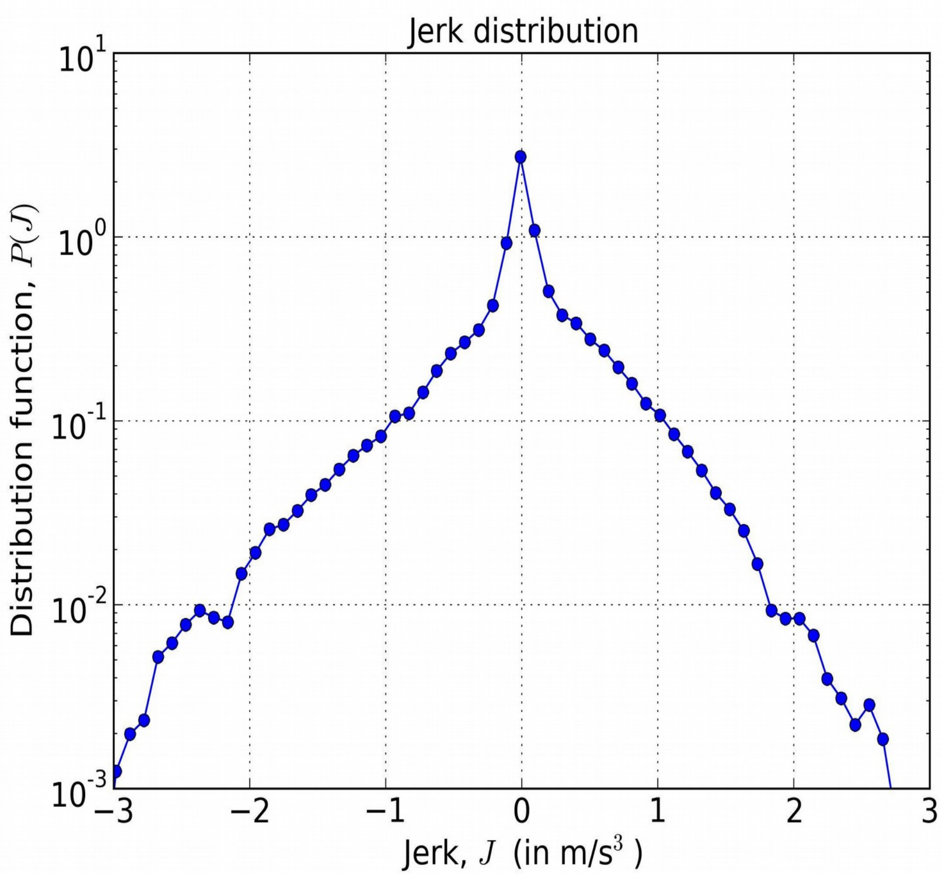
Data of real traffic



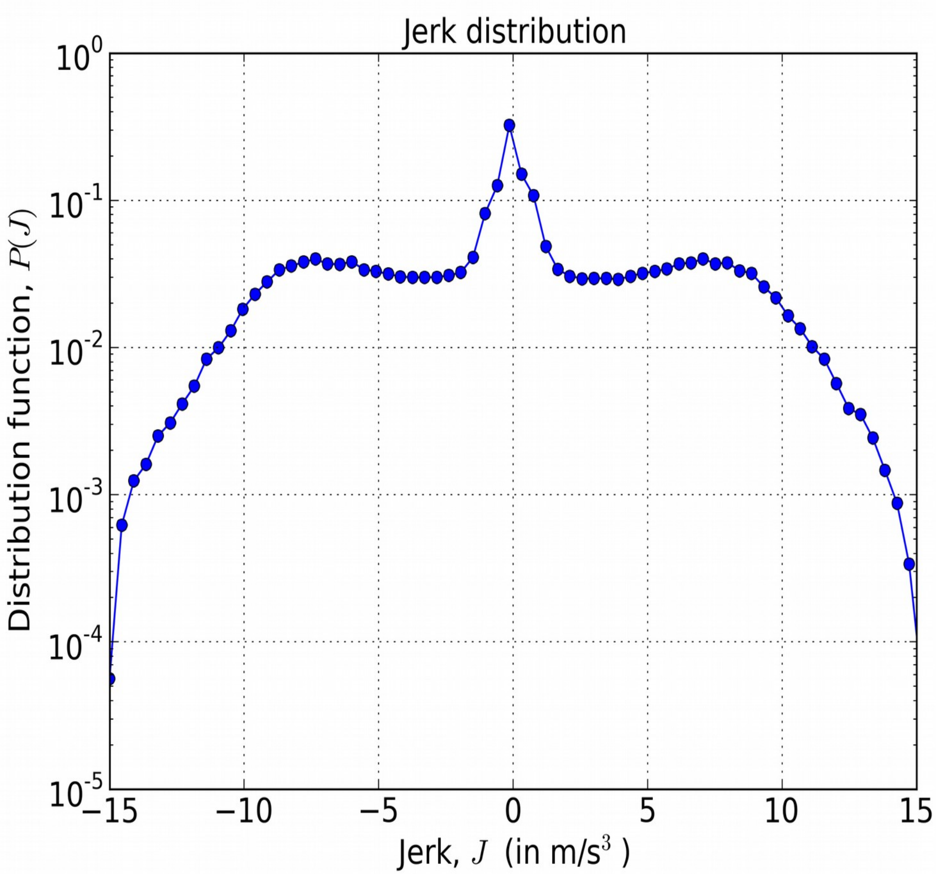
Jerk distribution: Car following setup

Jerk: $$j = \frac{da}{dt} = \frac{d^3x}{dx^3}$$

Subject with driving experience



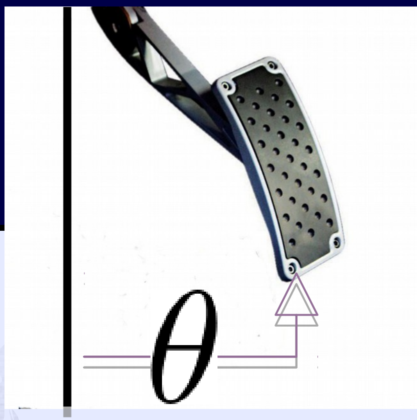
Subject without driving experience

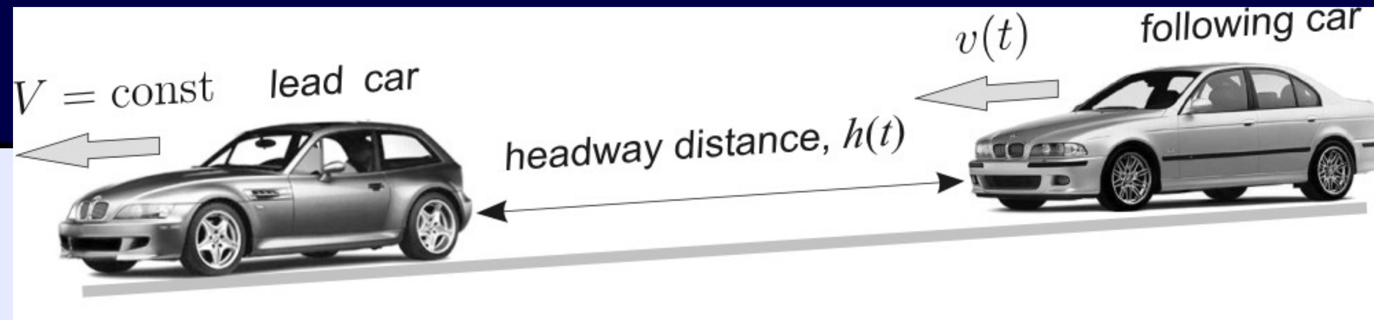


Conclusion to Driving Simulator Experiments

Hyporthesis: In order to describe car dynamics
the extended phase space is required.
It consists of the car ...

position $- x$	velocity $- v$	Phase variables of Newtonian mechanics
acceleration $- a$	jerk $- j$	Phase variables allowing for human actions





$$\frac{dh}{dt} = V - v$$

$$\frac{dv}{dt} = a$$

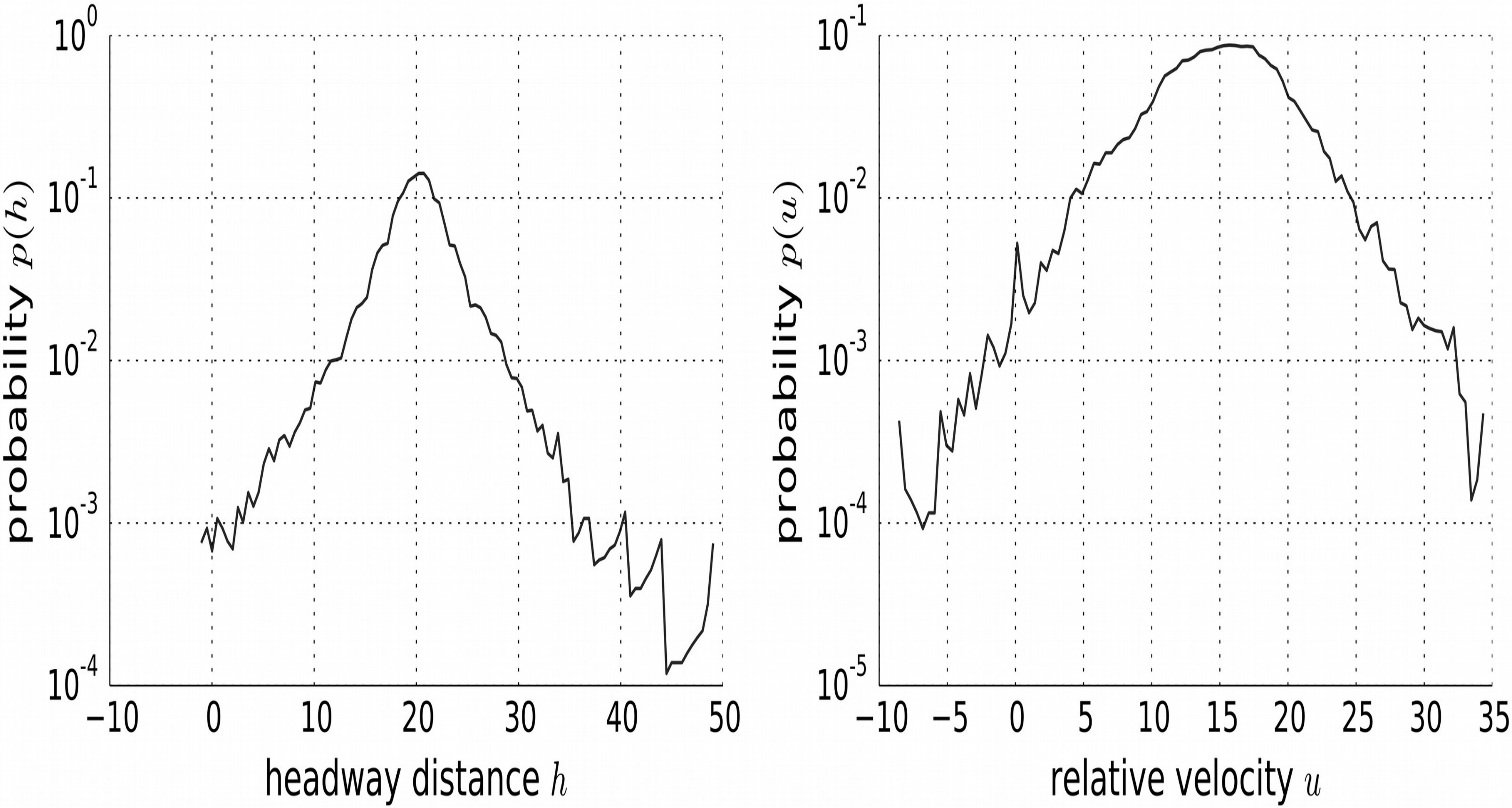
Kinematic relations

Car mechanical reaction

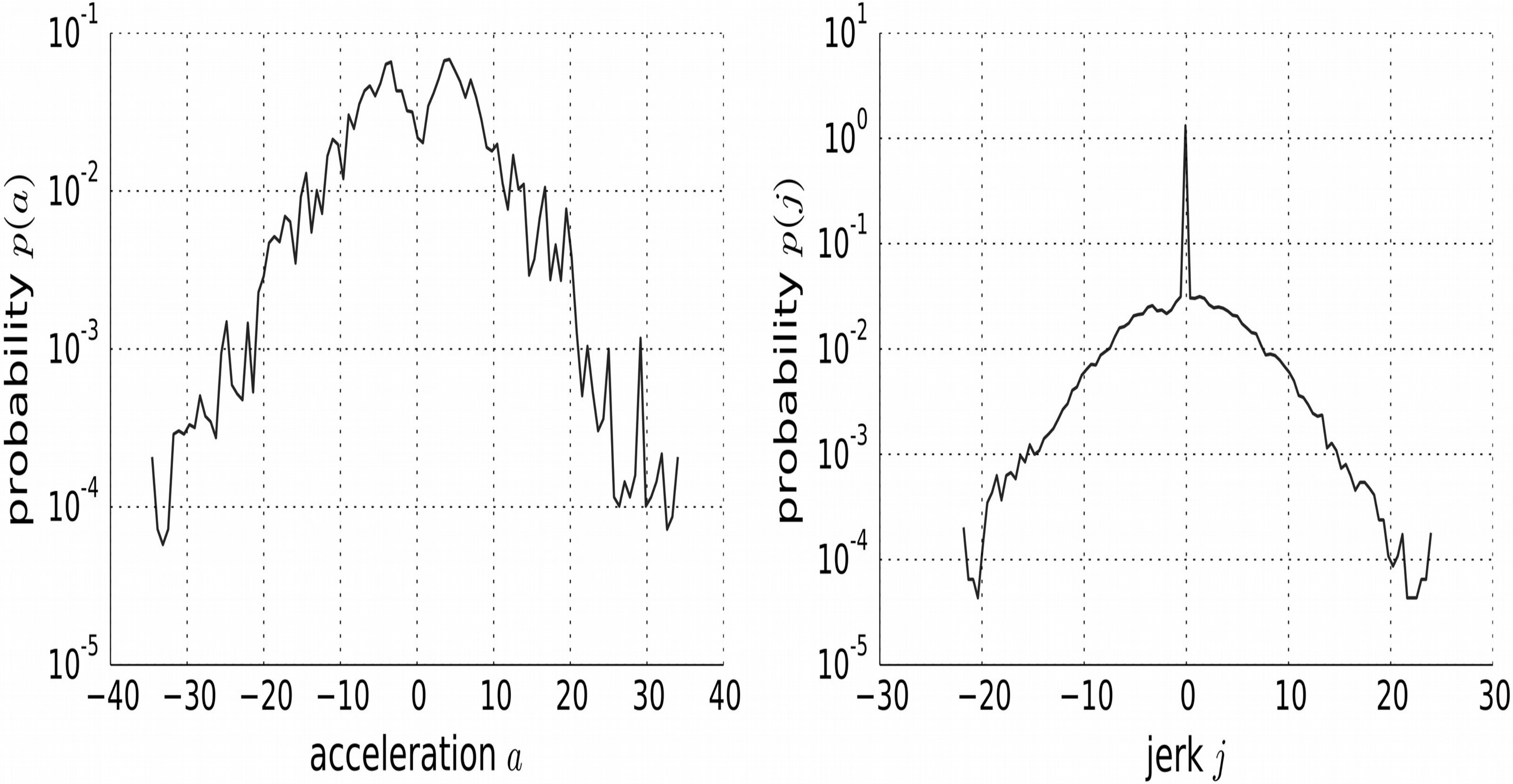
$$\tau_C \frac{da}{dt} = \theta - a$$

Human behavior

$$\tau_\theta \frac{d\theta}{dt} = \Omega(a - \theta)\left\{a_{\mathrm{opt}}(h, v) - a\right\} + \epsilon \xi(t)$$



probability $p(h)$
headway distance h
probability $p(u)$
relative velocity u



probability $p(a)$
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
−40 −30 −20 −10 0 10 20 30 40
acceleration a
probability $p(j)$
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
−30 −20 −10 0 10 20 30
jerk j